Physically Constrained Ensemble Kalman Filter

Erin Linebarger, Akil Narayan, Dongbin Xiu

Introduction

We present an improvement to the standard ensemble Kalman filter in the case where a physical constraint, e.g. conservation of energy, is present. Let S be some physical constraint formulated so that $S(v(j))=0$ is desired for all times j. The Kalman filter solution minimizes the variance of the state estimate $v(j)$, but in general does not satisfy the equality constraint. Thus a more accurate solution can be obtained by not only minimizing the variance of the state estimate, but by enforcing this constraint.

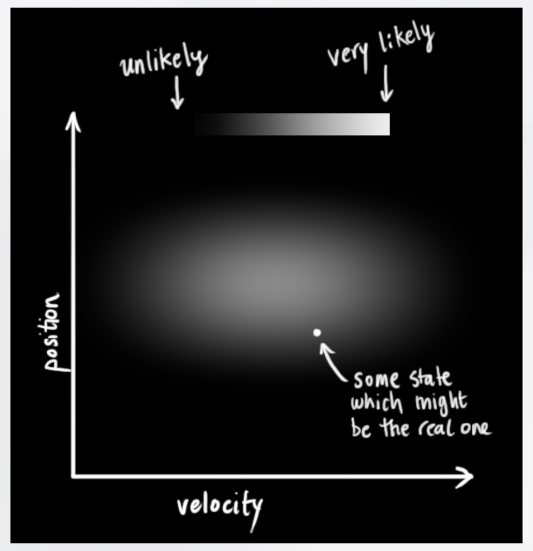
The Filtering Problem

- Stochastic or deterministic but complicated process
- Sequentially updated estimate of the state
- Uncertainty introduced through inherent stochasticity or lack of accuracy
- State as random variable
- Use Bayes formula to update probability distribution

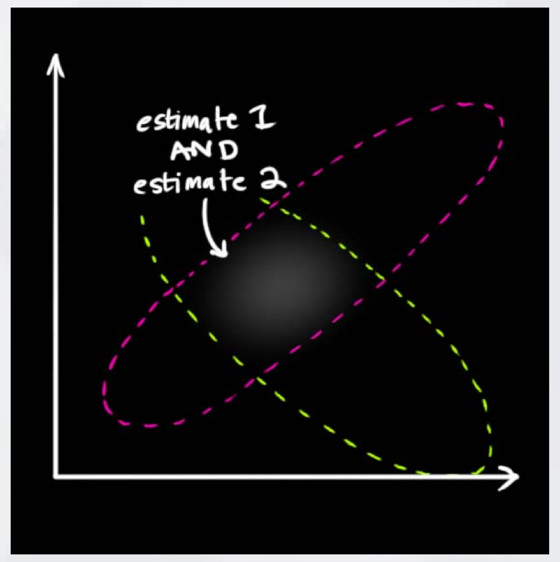
Modeling Problem

Inherent Uncertainty

Uncertainty estimation



Filtering Update



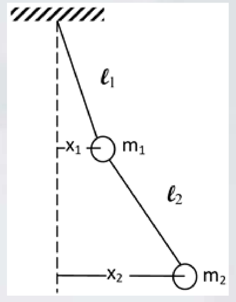
The Importance of Constraints

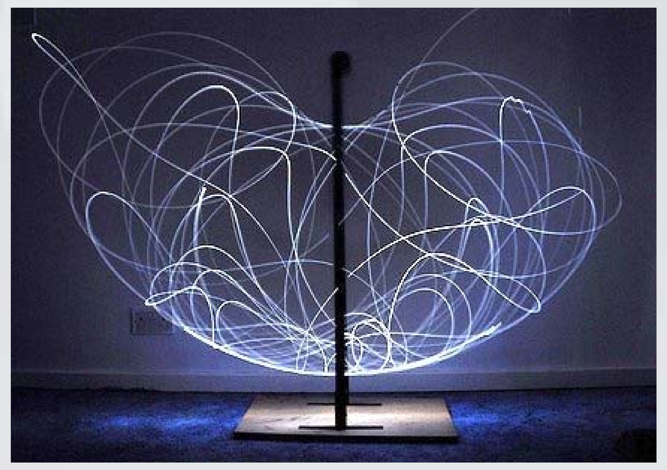
A filtered solution does not necessarily respect physical constraints, whether in the form of an equality or an inequality. Therefore by only choosing solutions that satisfy the constraints, we can often obtain a more accurate solution, since the true solution would satisfy such constraints.

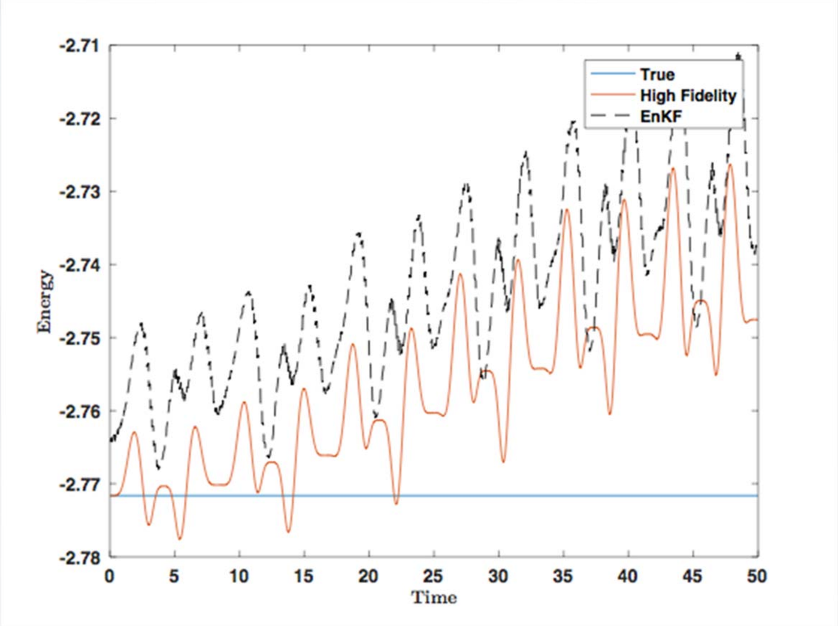
Physical constraints, e.g. conservation of energy, are important for simulation science: A simulation that does not adhere to these constraints is an unphysical prediction. A filtering update 'pushes' the system state towards observed data. However, this 'push' may deflect the state to a configuration that violates physical constraints.

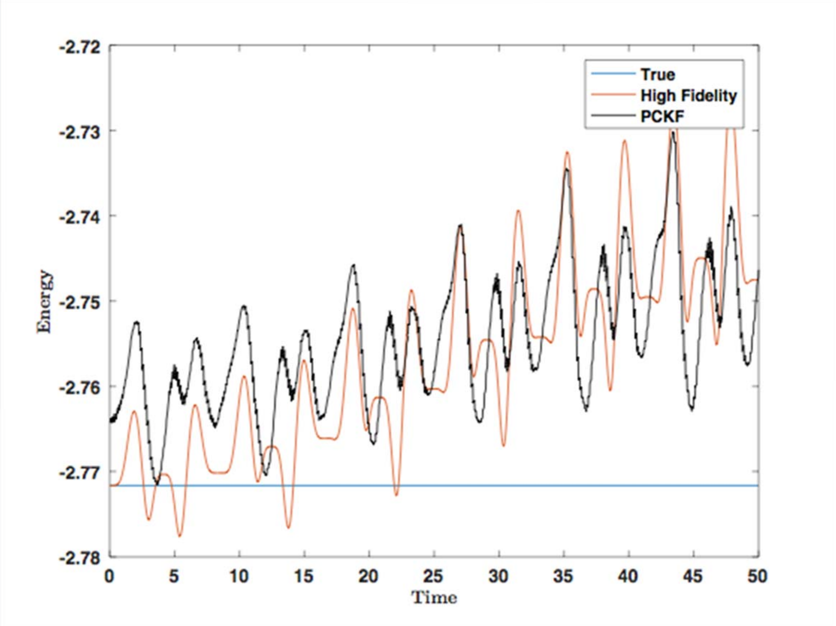
A standard Kalman filter, therefore, tends to produce simulation results that do not obey natural constraints, and hence is an unphysical prediction.

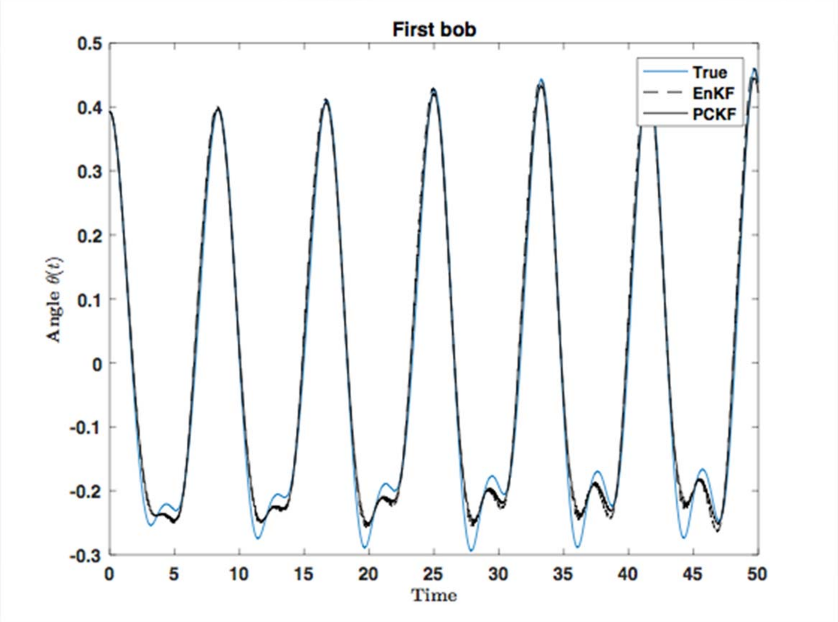
Double Pendulum

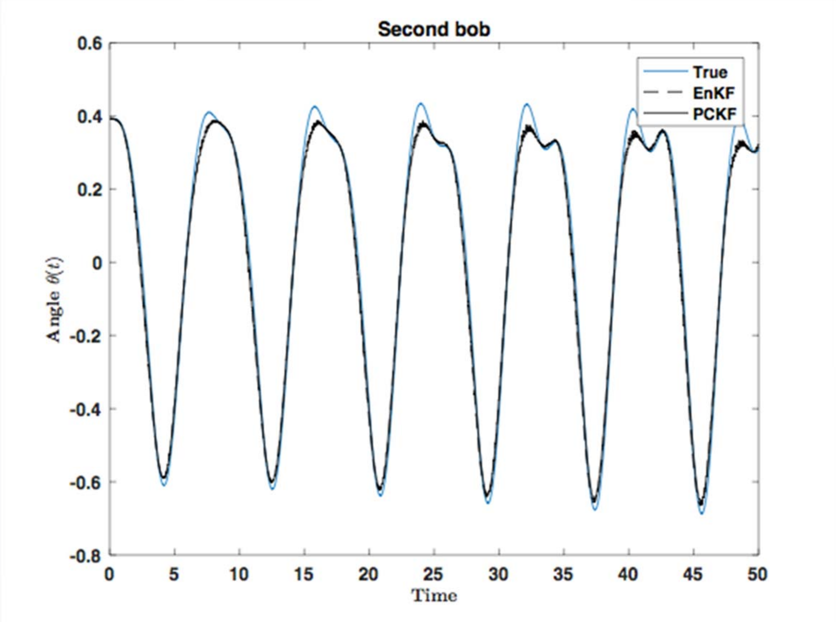


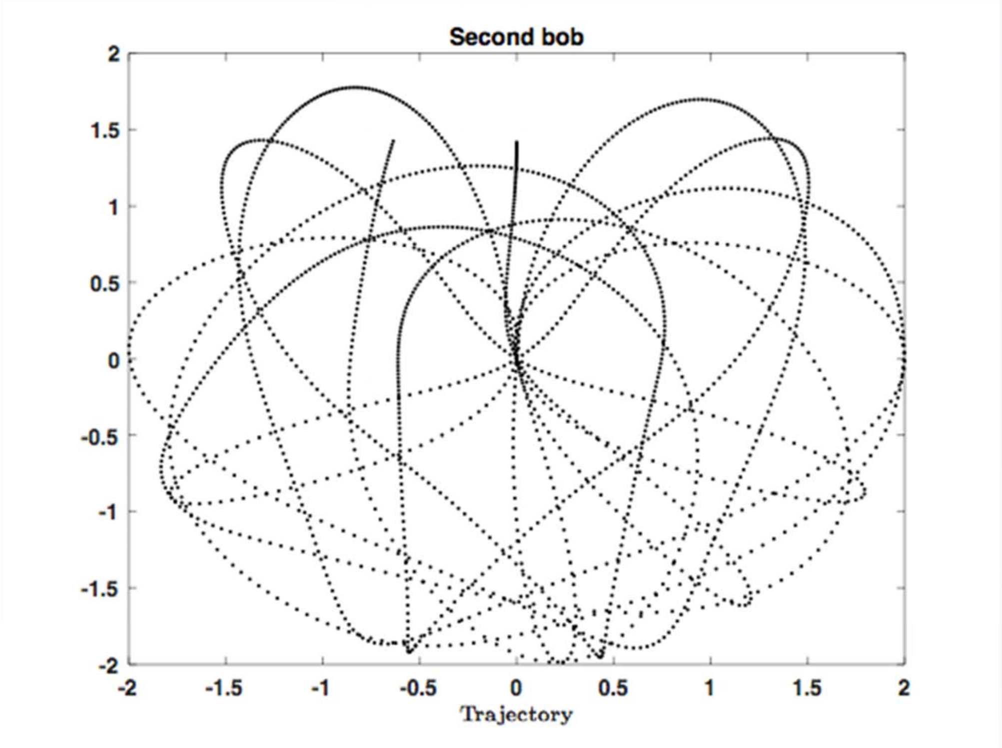


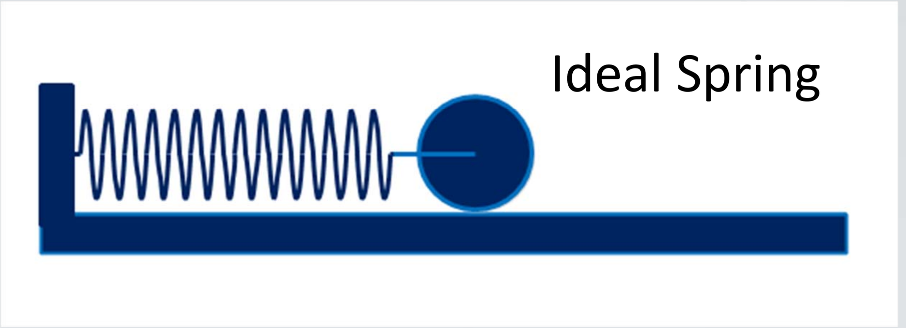




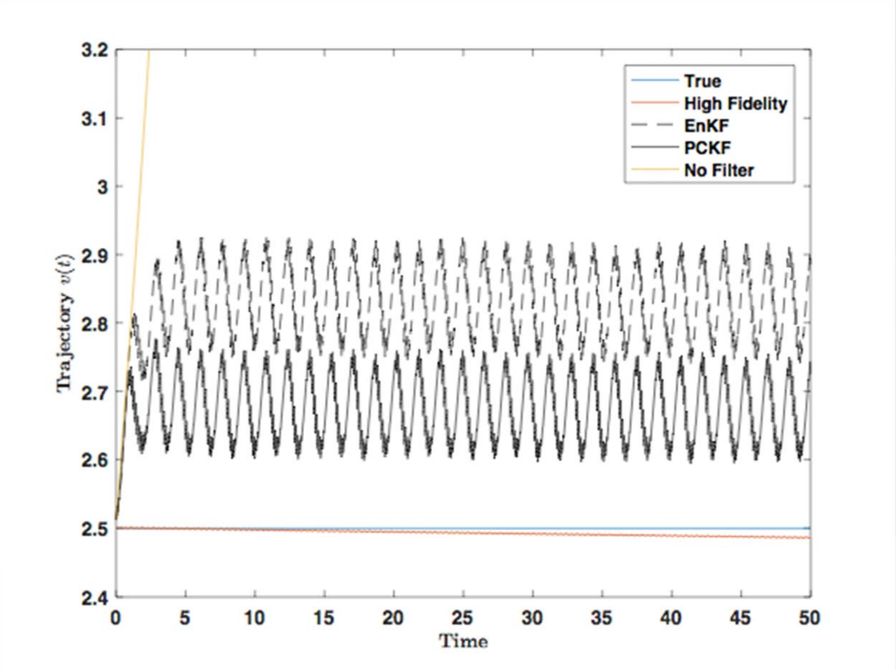




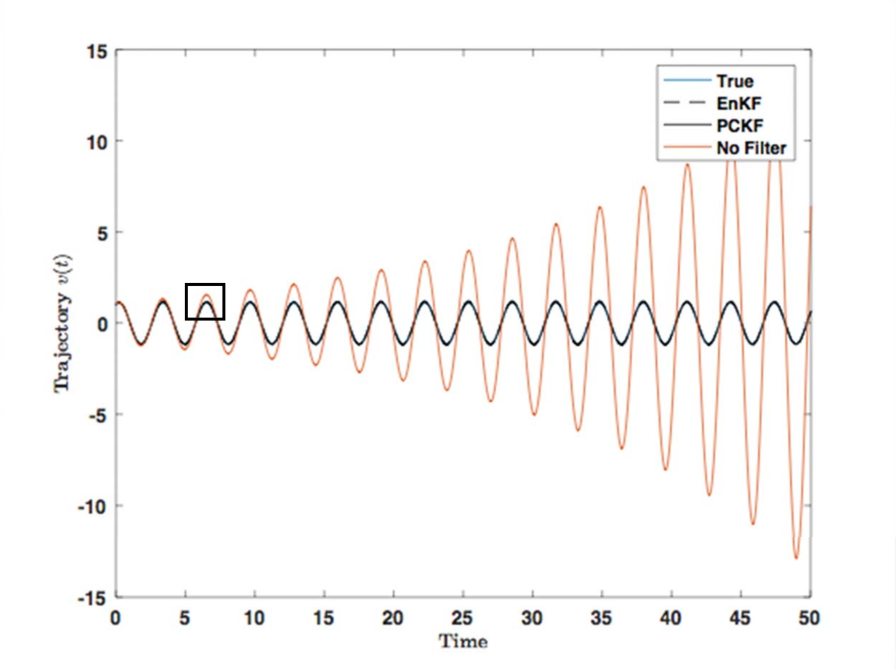


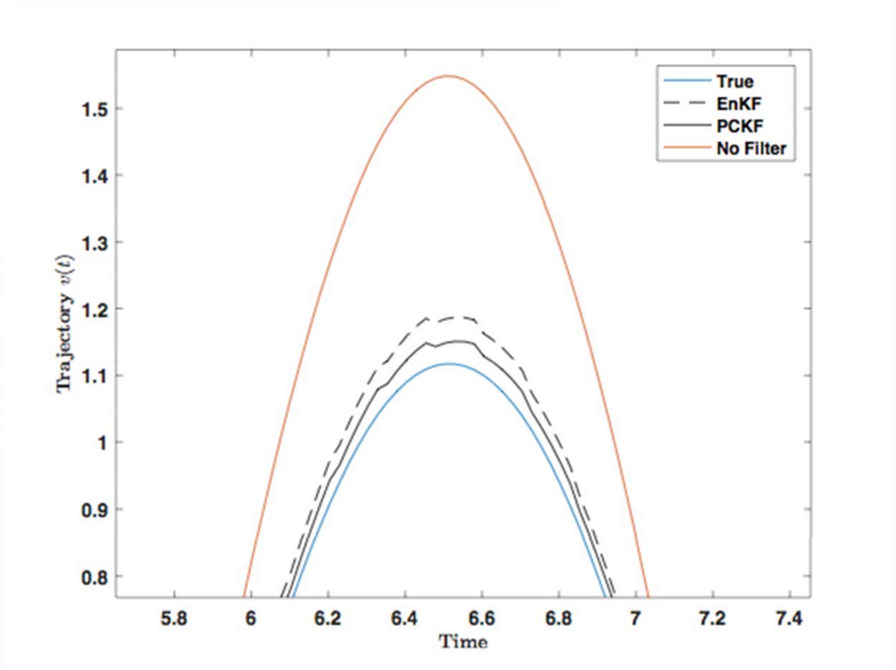


Spring Energy



Spring Trajectory





References

Law, Kody, Andrew Stuart, and Konstantinos Zygalakis. *Data Assimilation: A Mathematical Introduction*. Switzerland: Springer International Publishing, 2015.
Kalman Filter robot images and graphs by Tim Babb from http://www.bzarg.com/p/how-a-kalman-filter-works-in-pictures/, Accessed Nov 6, 2016.
Double Pendulum from http://www.chegg.com/homework-help/questions-and-answers/assuming-small-amplitudes-set-differential-equations-motion-double-pendulum-using-coordina-q1670907, Accessed Nov 6, 2016.
LED Double Pendulum By George Ioannidis (Own work) [CC BY 3.0 (http://creativecommons.org/licenses/by/3.0)], via Wikimedia Commons