The
Mutual Information Diagram
for
Uncertainty Visualization

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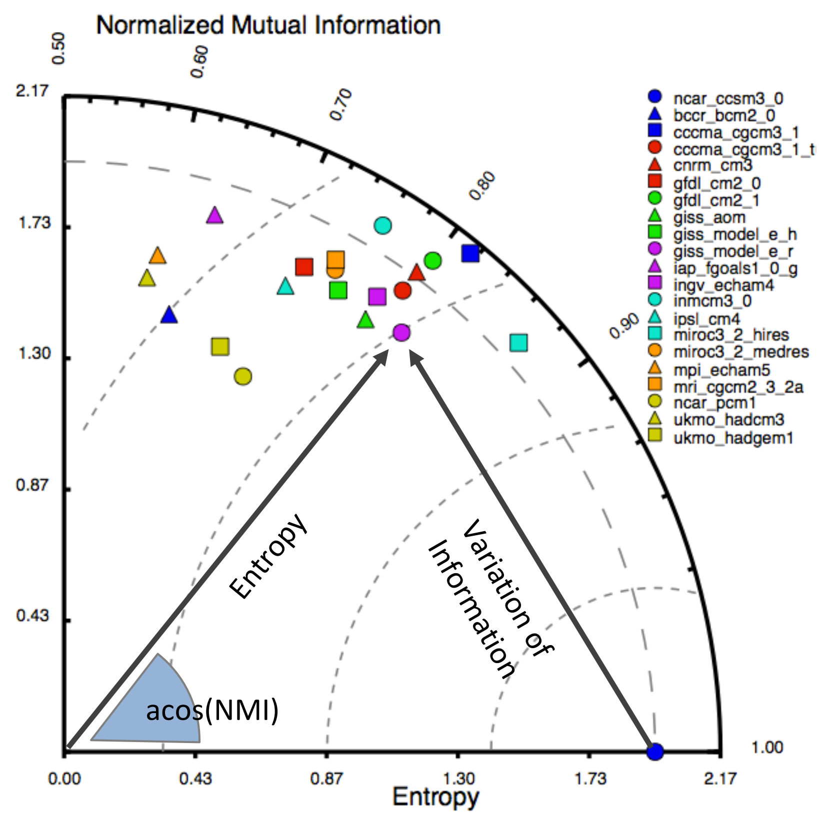
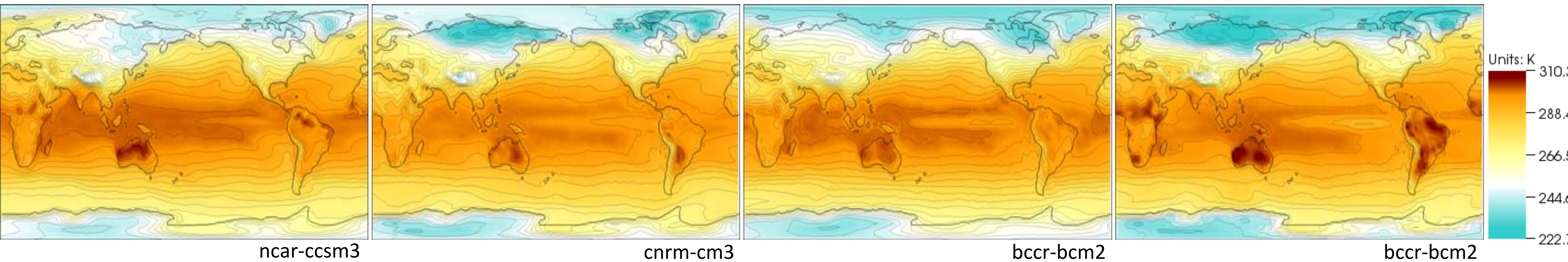
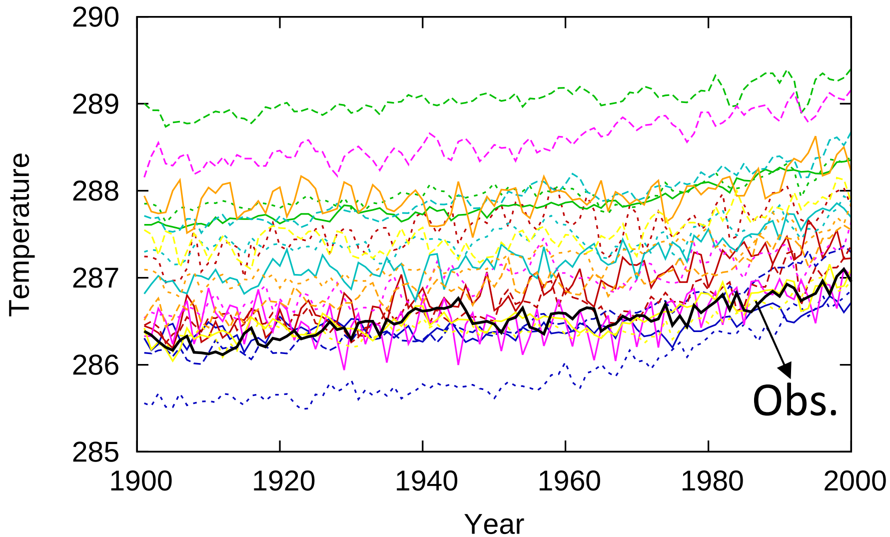


This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Visualization should help compare models with observations



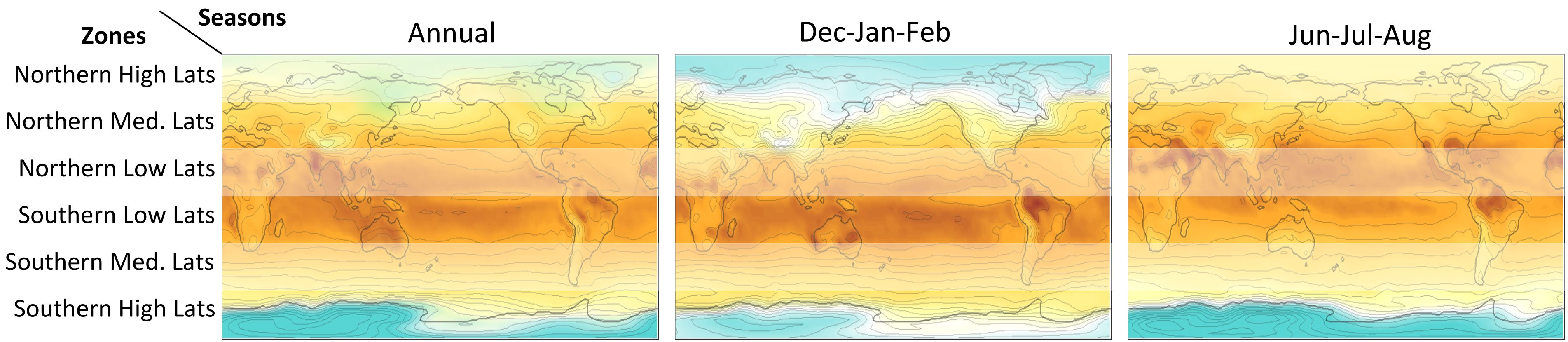


Average annual temperature 1900-2000 as predicted by various climate models.

Which model is more **similar** to a reference model or observations?

Trend plots often do not expose these aspects.

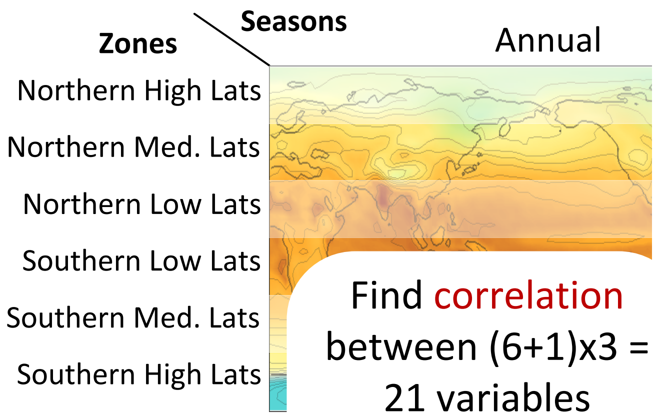
Visualization should help find correlations of similar outputs – important for uncertainty quantification



Divide ensembles in 6 latitude zones and 3 temporal averages

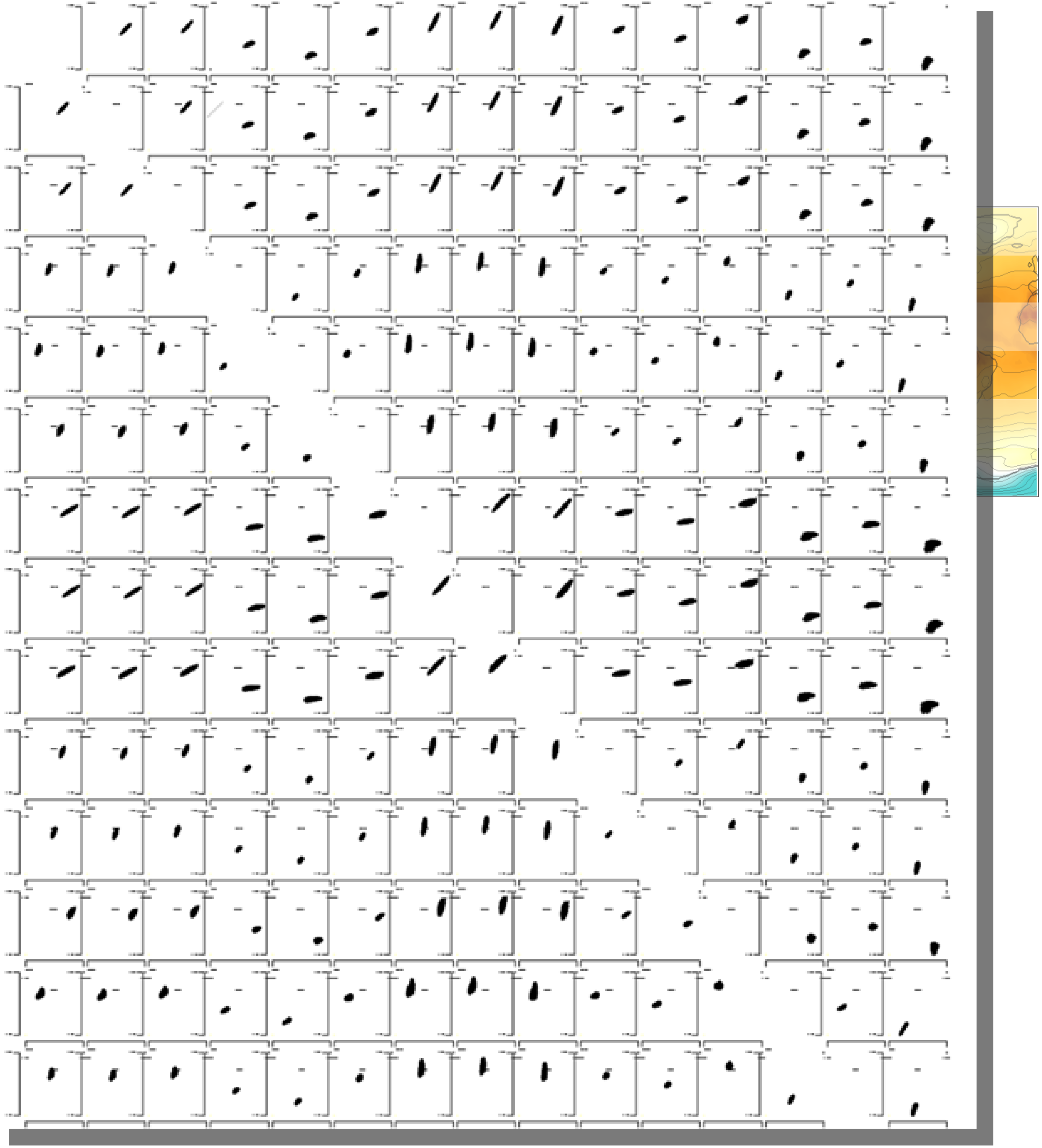
- Are there correlations across seasons or latitudes?
- Are there large discrepancies in the different outputs?

Visualization shoul
importa

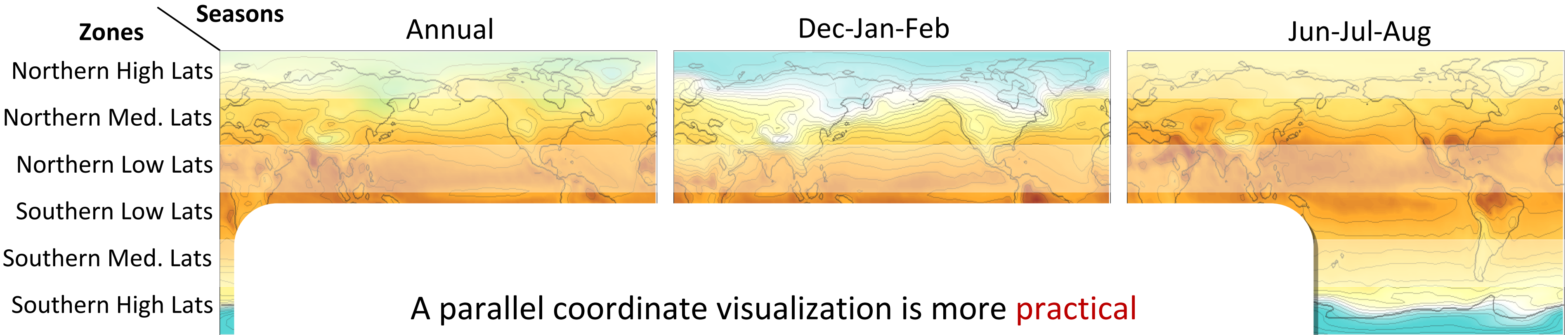


Find correlation between (6+1)x3 = 21 variables

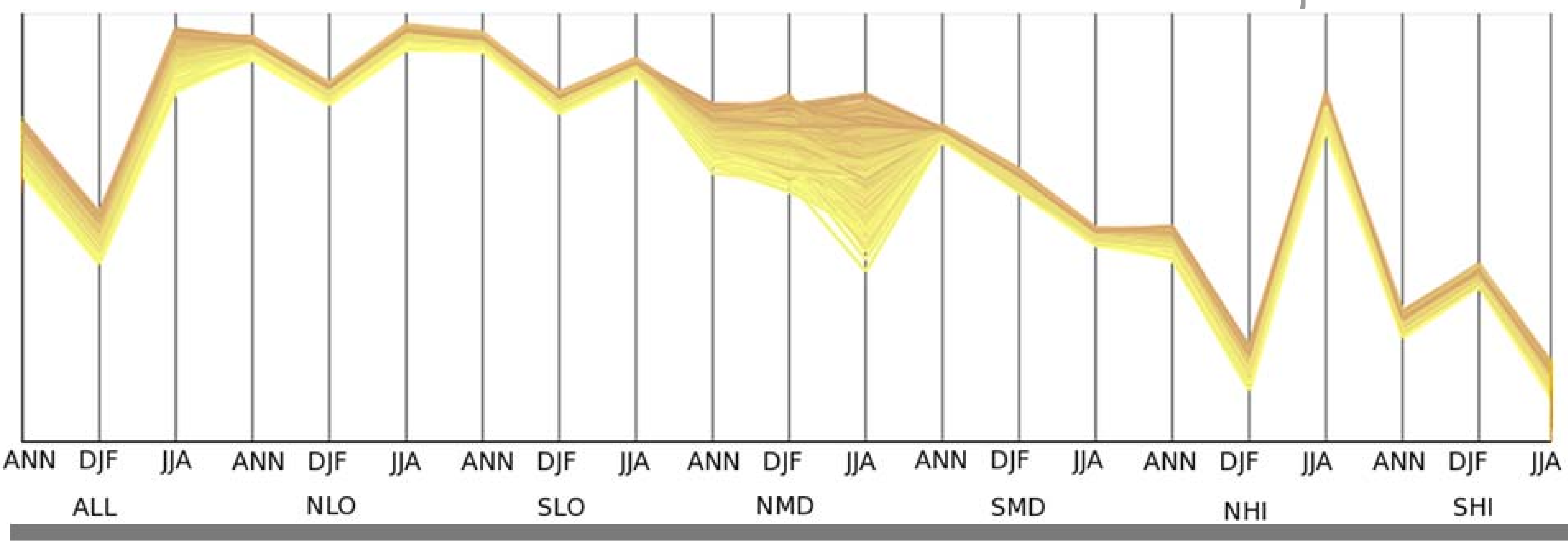
A scatterplot matrix becomes impractical for many outputs



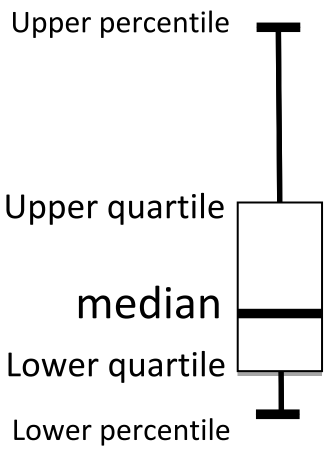
Visualization should help find correlations of similar outputs – important for uncertainty quantification

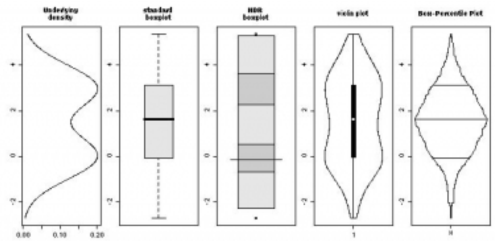


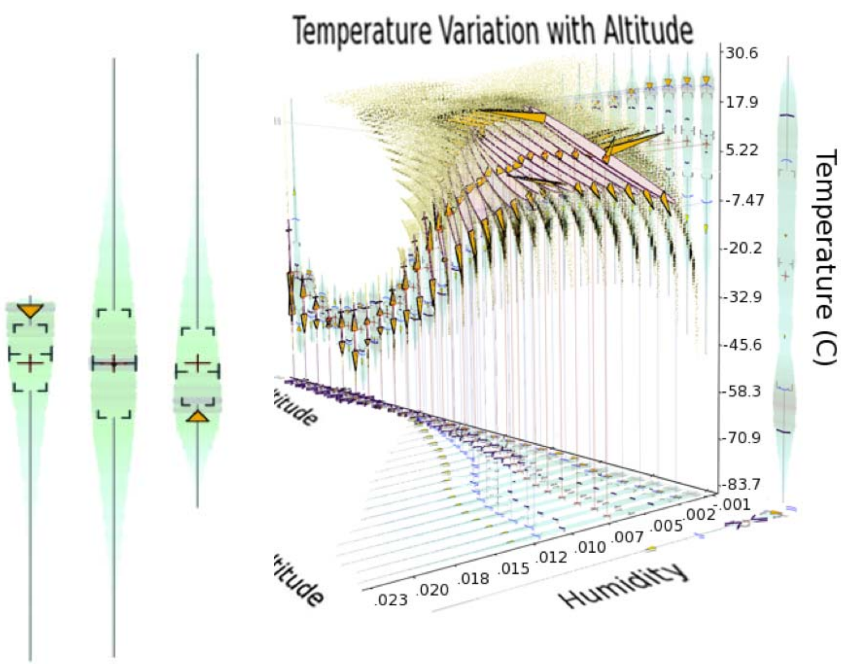
A parallel coordinate visualization is more practical
But only certain pairwise comparisons are possible



Visual Summaries



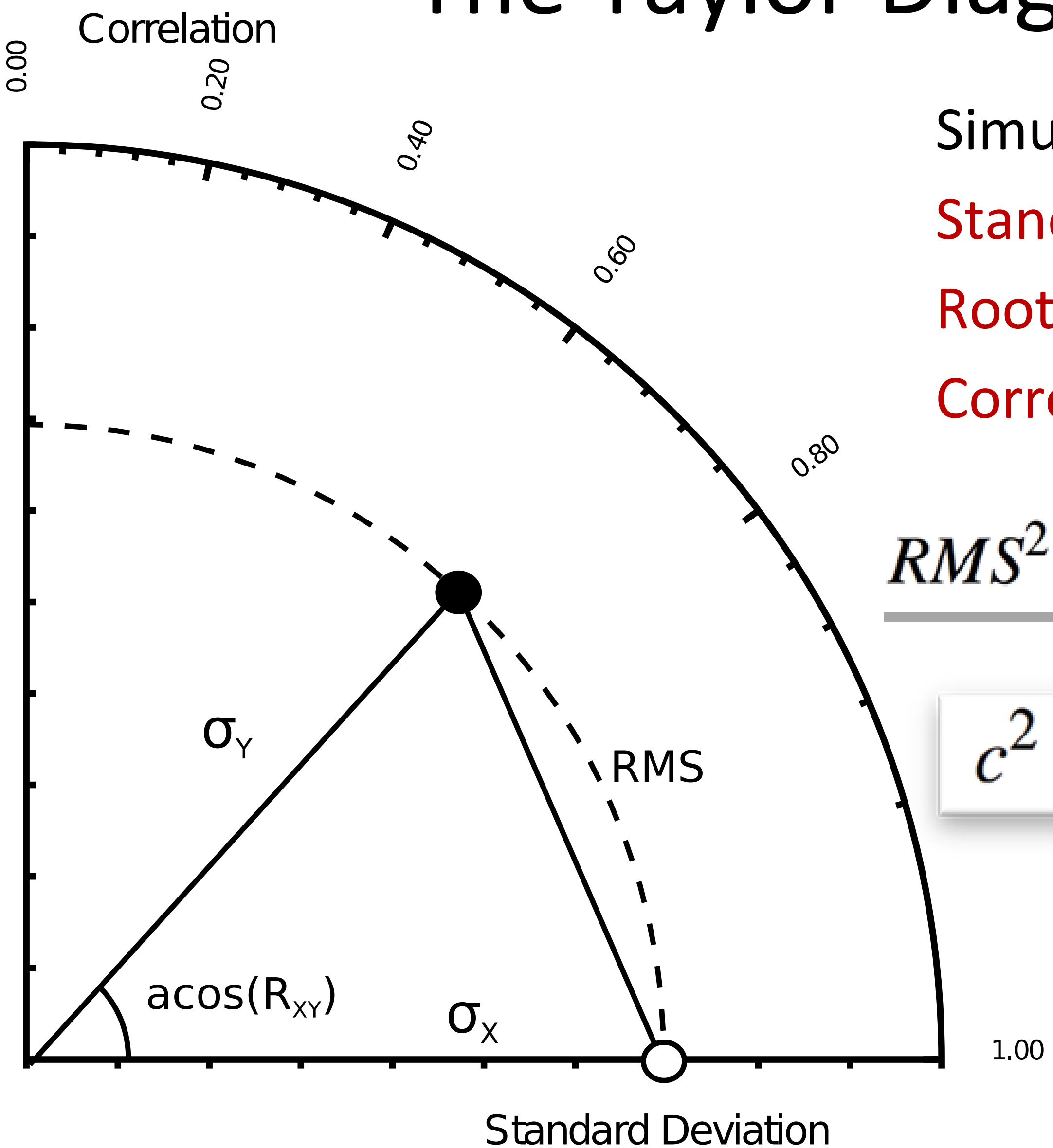




Potter et al. (Eurovis'2010)

- Represent directly summary quantities, e.g., mean, standard deviation, entropy.
- Box-plots and their many variants
- One plot per ensemble may result in clutter
- Visualizing several statistics simultaneously in a metric space: Taylor diagram

The Taylor Diagram

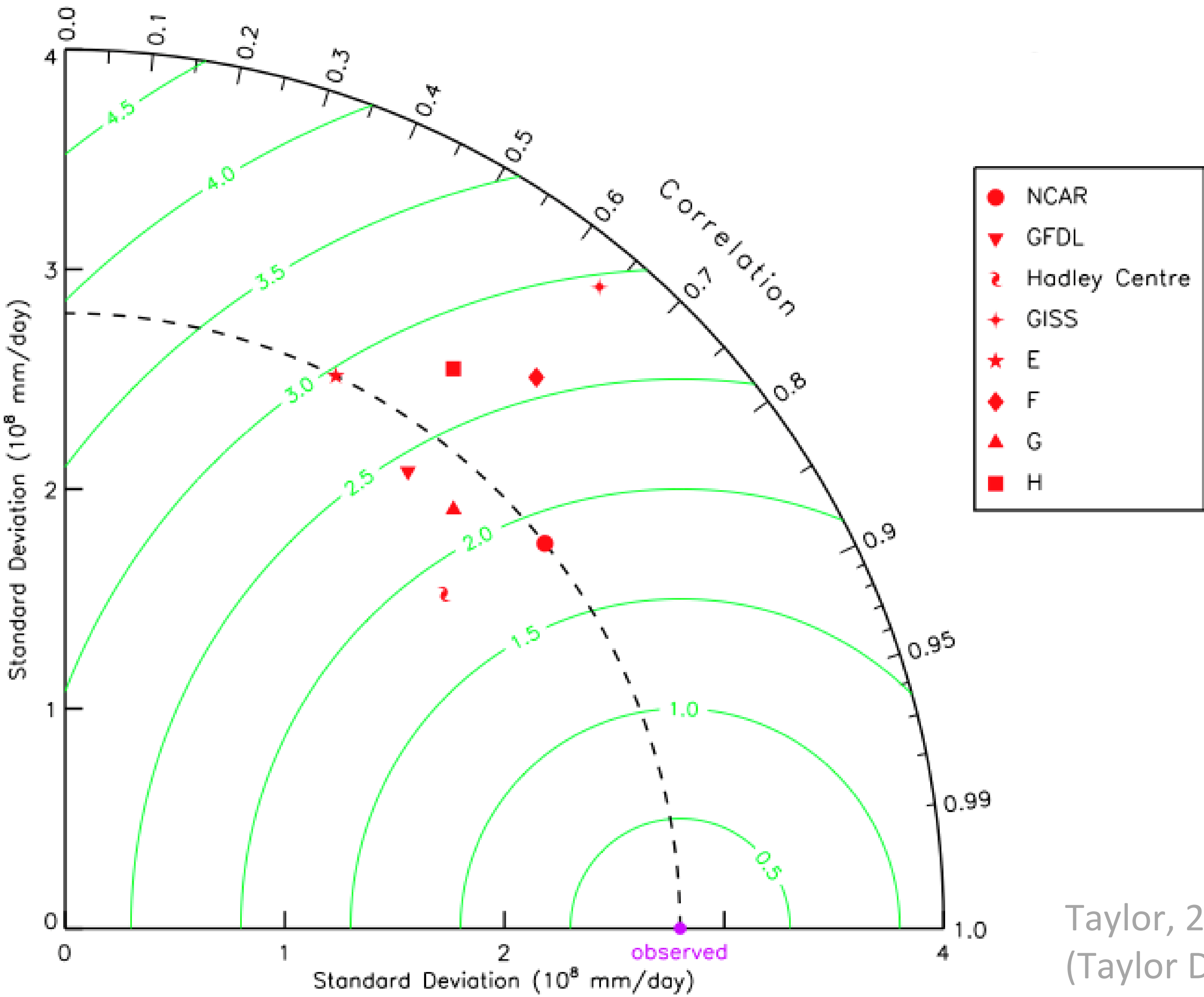


Simultaneously plots
Standard deviation,
Root Mean Square Error and
Correlation R.

$$RMS^2 = \sigma_X^2 + \sigma_Y^2 - 2\sigma_X \sigma_Y R_{XY}$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

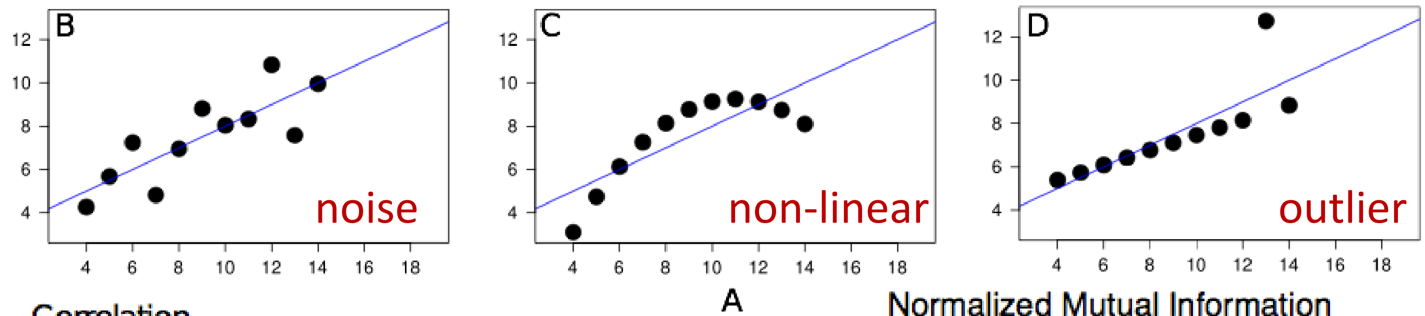
Applications of the Taylor Diagram

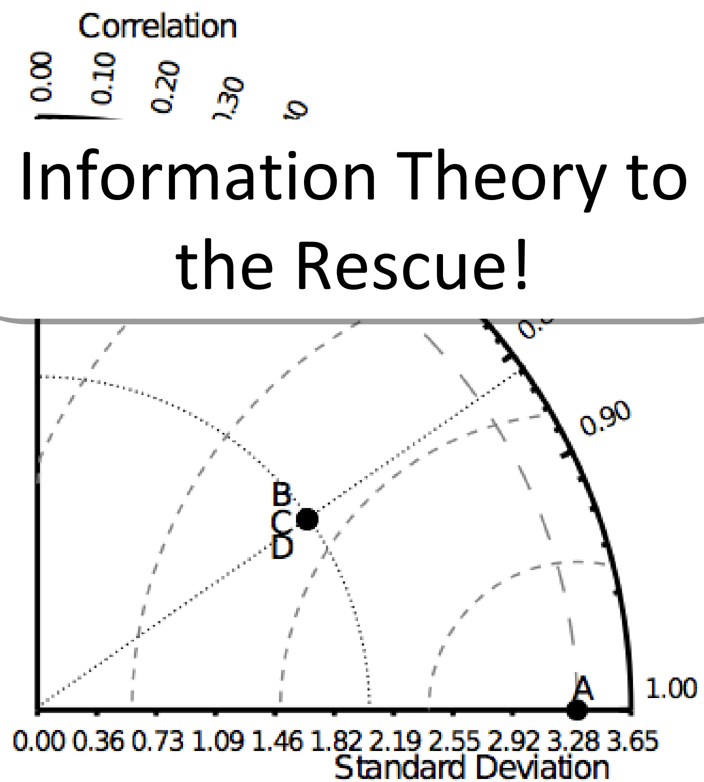


Taylor, 2005
(Taylor Diagram Primer)

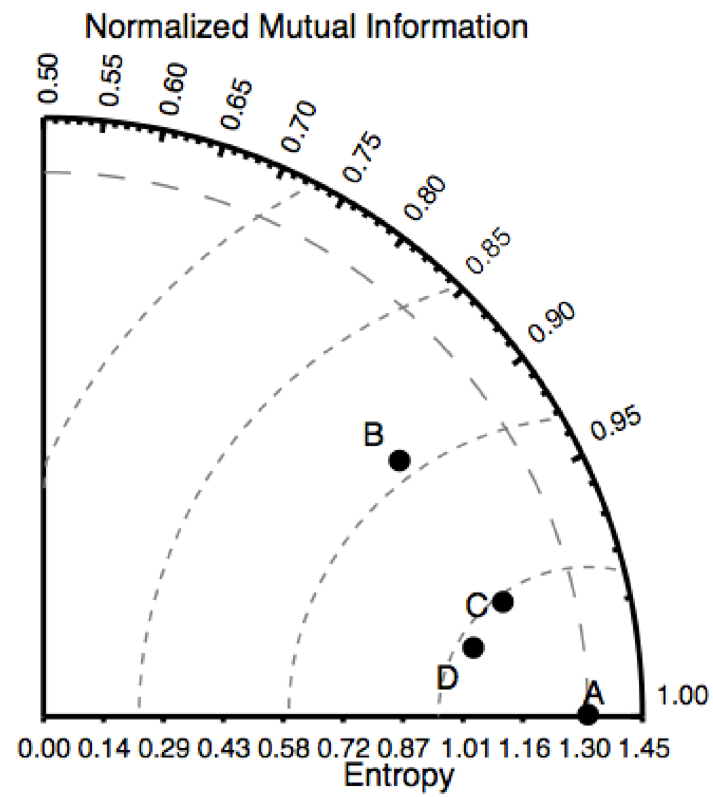
Anscombe's Trio

Variables B,C,D: same standard deviation and same correlation w.r.t. A

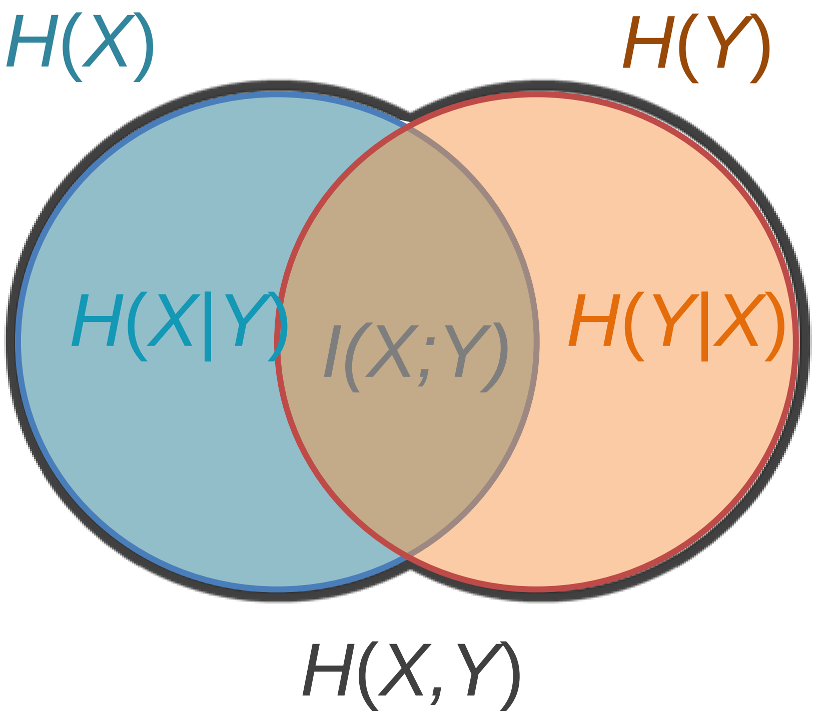




Information Theory to the Rescue!



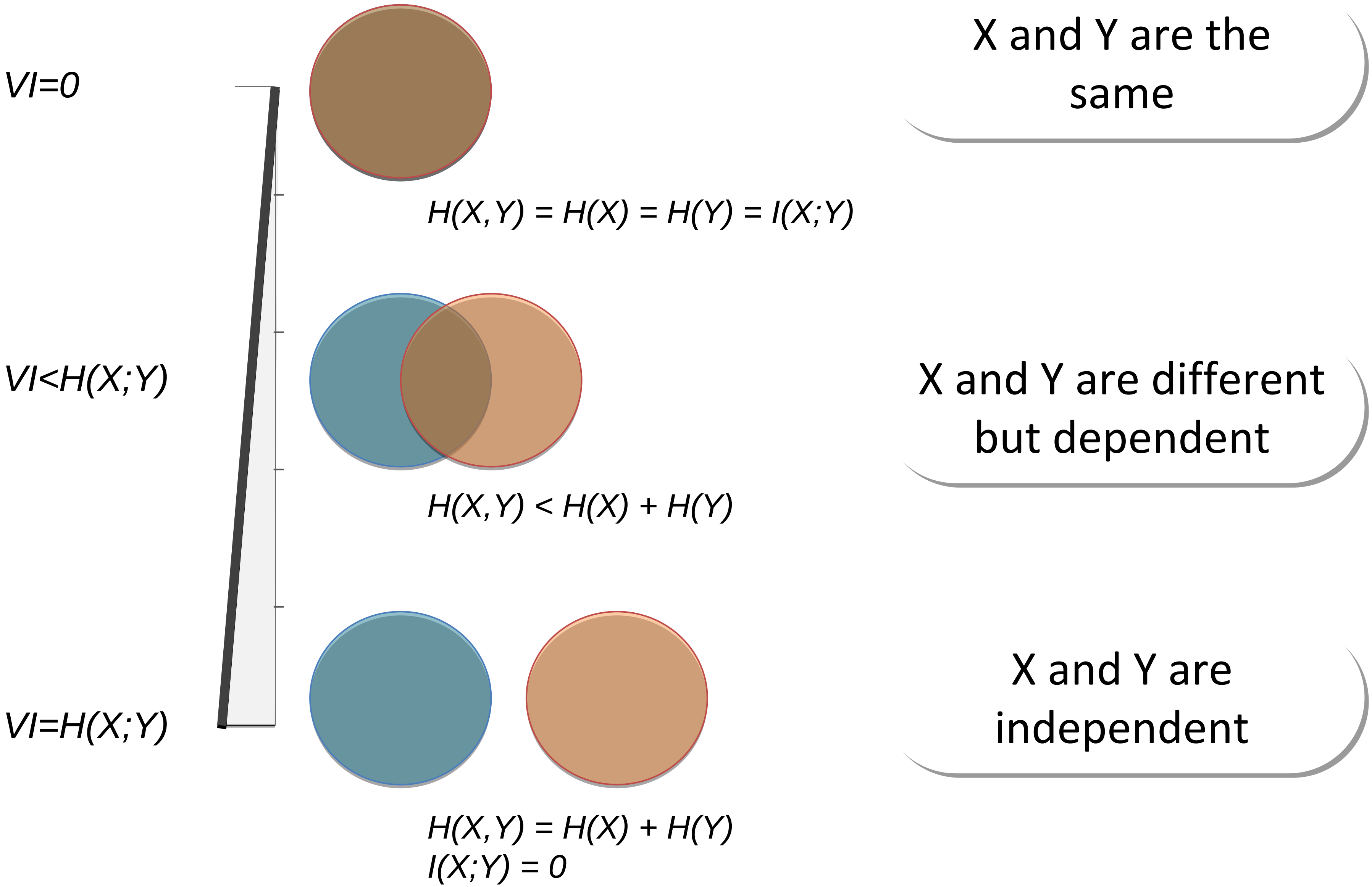
Information Theory Primer



$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

- Entropy H(X)
  - Measure of information uncertainty of X
- Joint Entropy H(X,Y)
  - Uncertainty of X,Y
- Conditional Entropy H(X|Y)
  - Uncertainty of X given that I know Y
- Mutual Information I(X;Y)
  - How much knowing X reduces the uncertainty of Y

Variation of Information $VI = H(X \mid Y) + H(Y \mid X)$



The Variation of Information VI: a measure of distance in information theory

$$VI(X,Y) = H(X) + H(Y) - 2I(X;Y)$$

$$RVI = \sqrt{VI}$$

$$h_X = \sqrt{H(X)}$$

$$h_Y = \sqrt{H(Y)}$$

$$RVI(X,Y)^2 = h_X^2 + h_Y^2 - 2I(X;Y)$$

$$RVI(X,Y)^2 = h_X^2 + h_Y^2 - 2h_X h_Y \frac{I(X;Y)}{h_X h_Y}$$

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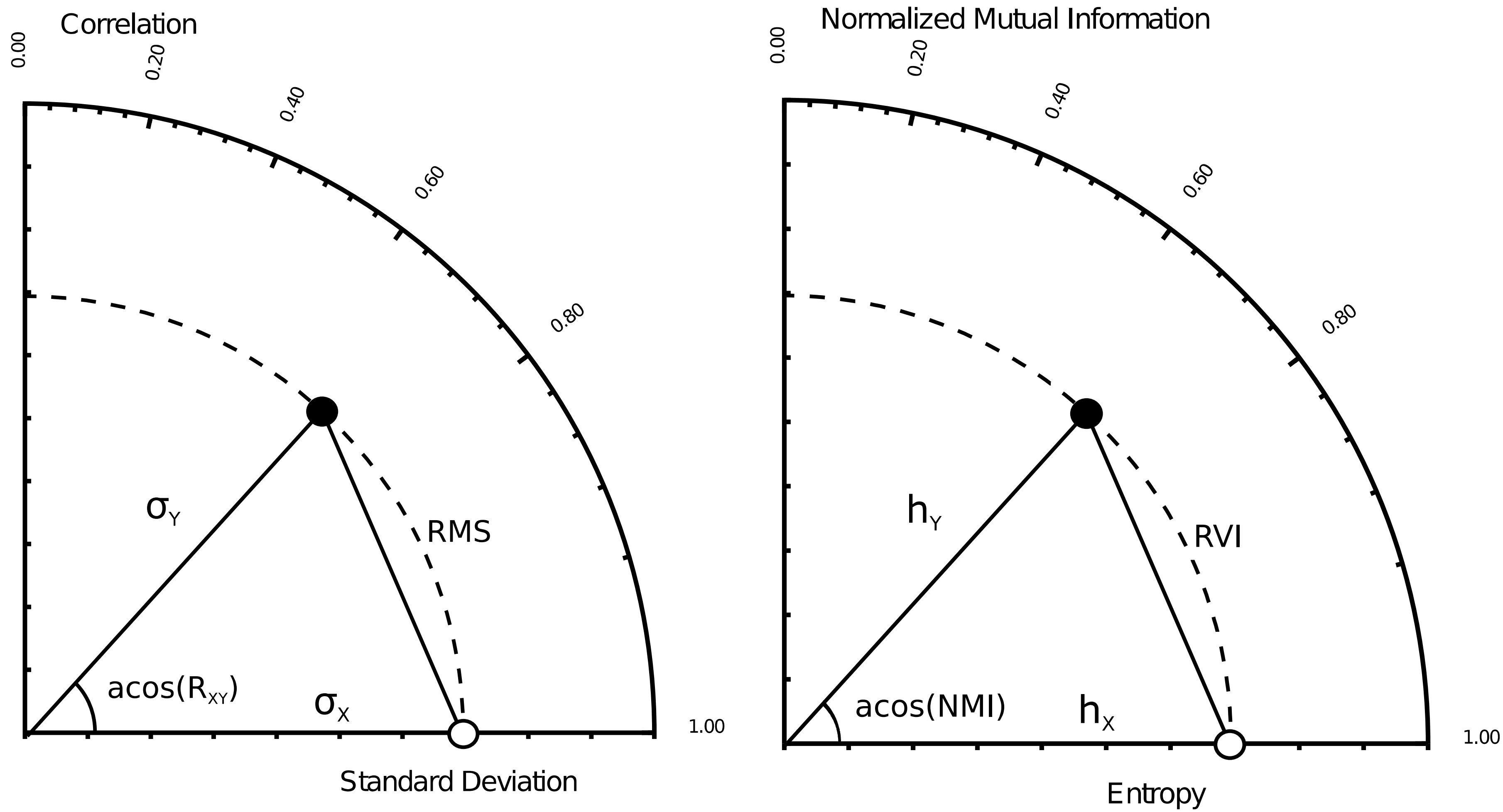
Normalized Mutual Information (NMI)

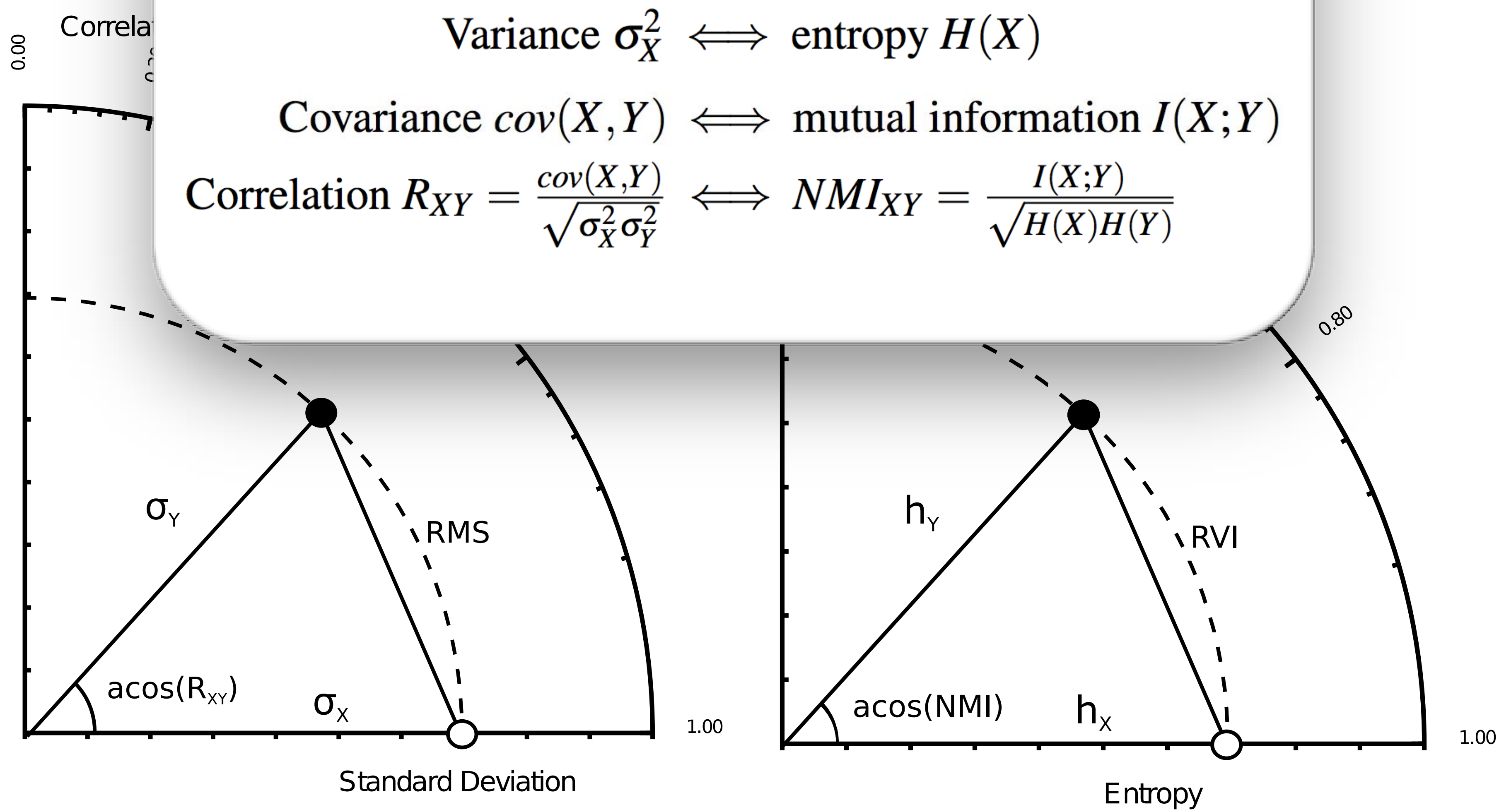
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

RVI Diagram

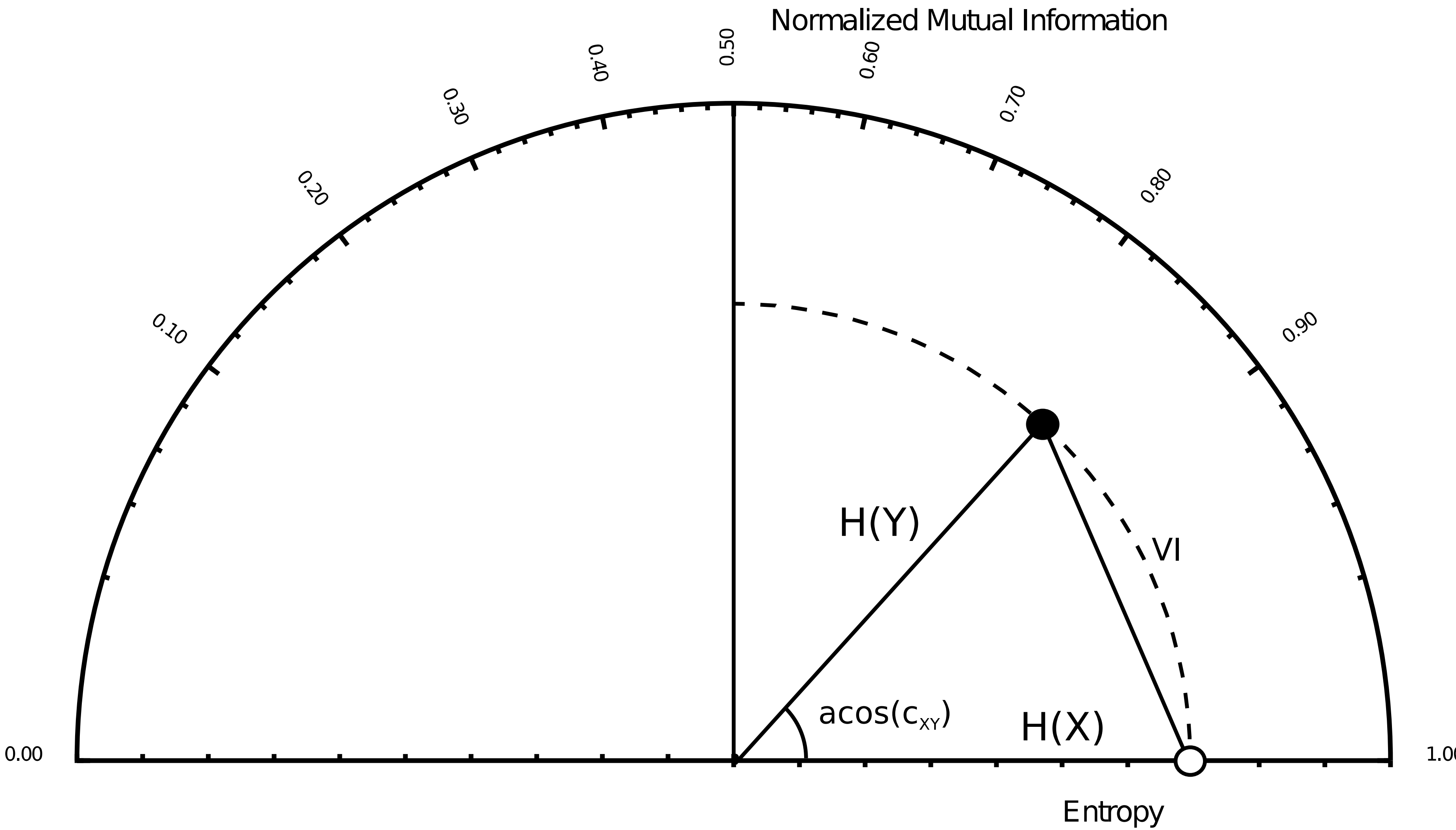

Correlation
0.00
0.20
0.40
0.60
0.80
1.00
σY
RMS
acos(RXY)
σX
Standard Deviation
Normalized Mutual Information
0.00
0.20
0.40
0.60
0.80
1.00
hY
RVI
acos(NMI)
hX
Entropy

Equivalences

Statistics	$\Longleftrightarrow$	Information Theory
$\mathrm{RMS}(X,Y)$	$\Longleftrightarrow$	RVI $\sqrt{VI(X,Y)}$
Variance σ_X^2	$\Longleftrightarrow$	entropy $H(X)$
Covariance $cov(X,Y)$	$\Longleftrightarrow$	mutual information $I(X;Y)$
Correlation $R_{XY} = \frac{cov(X,Y)}{\sqrt{\sigma_X^2 \sigma_Y^2}}$	$\Longleftrightarrow$	$NMI_{XY} = \frac{I(X;Y)}{\sqrt{H(X)H(Y)}}$

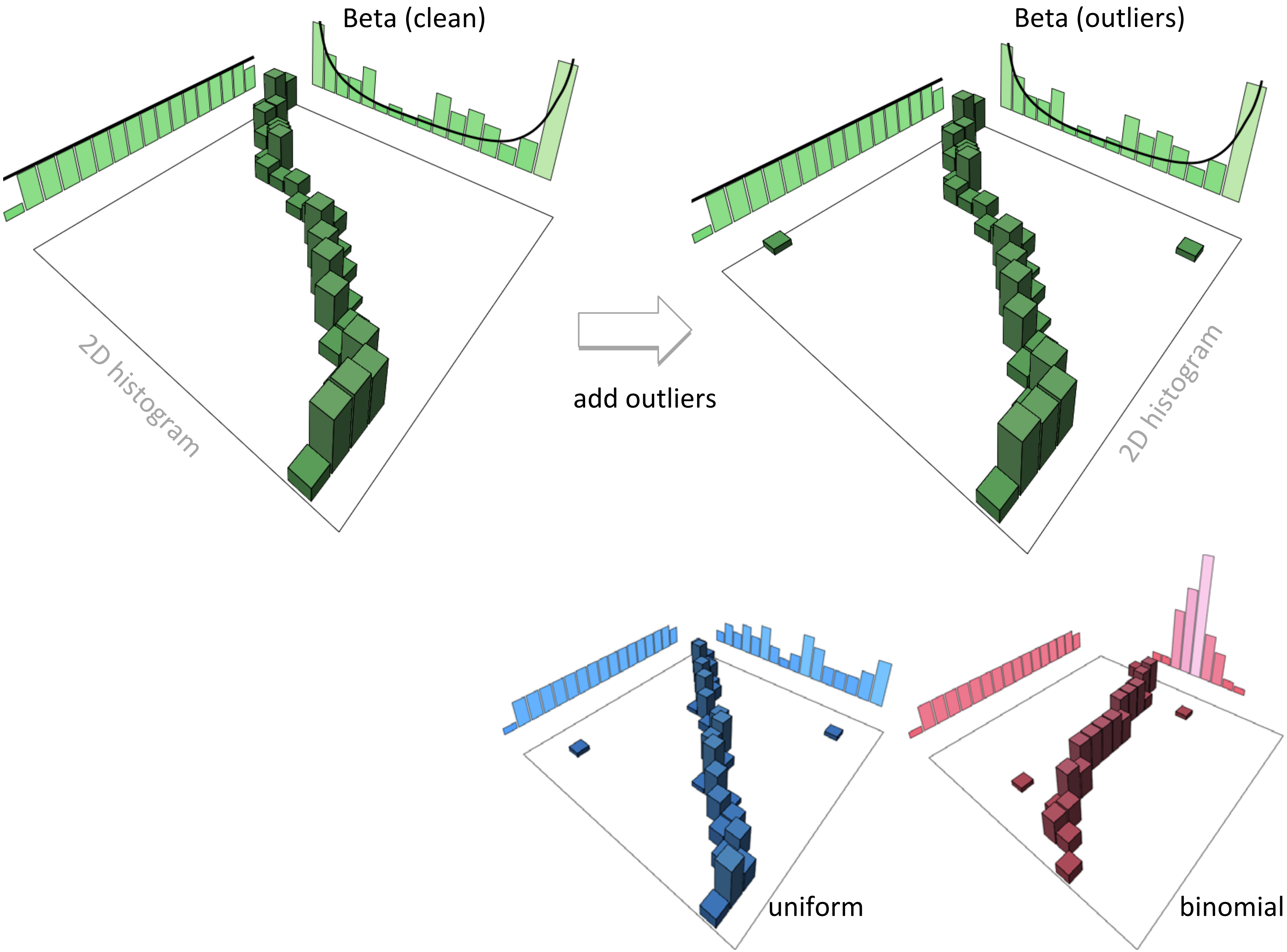


VI Diagram



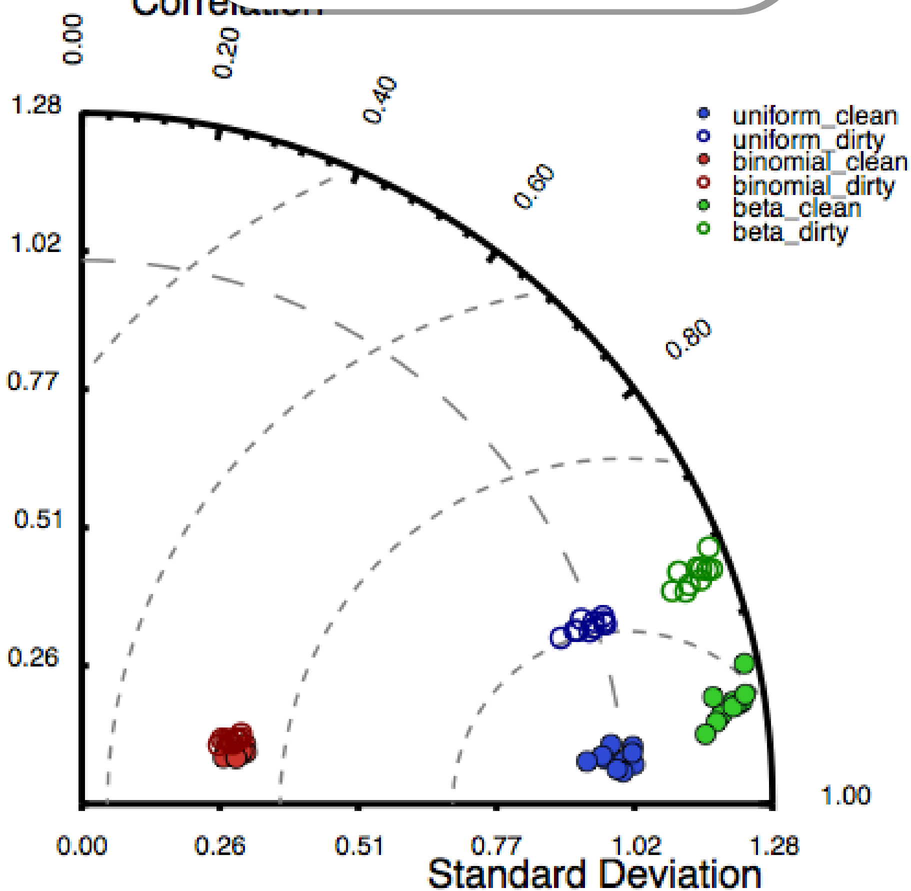
$$VI(X,Y)^2 = H(X)^2 + H(Y)^2 - 2H(X)H(Y)c_{XY}$$

Experiment of 2D distributions with outliers

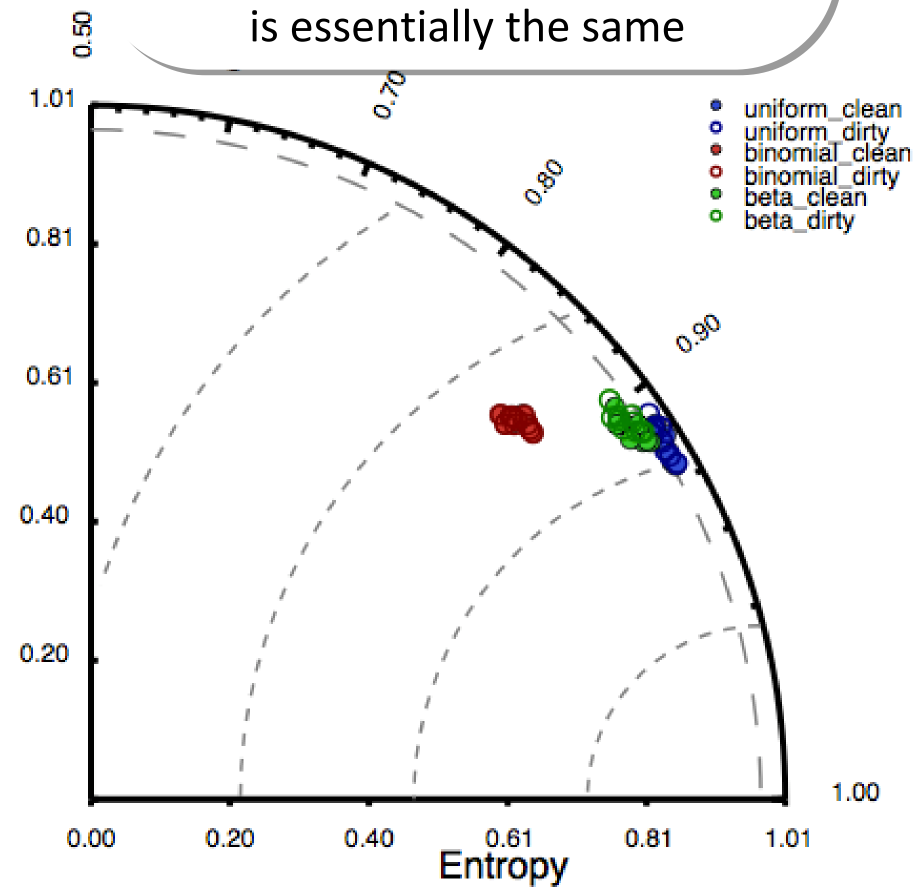


MI diagram is more resilient to outliers

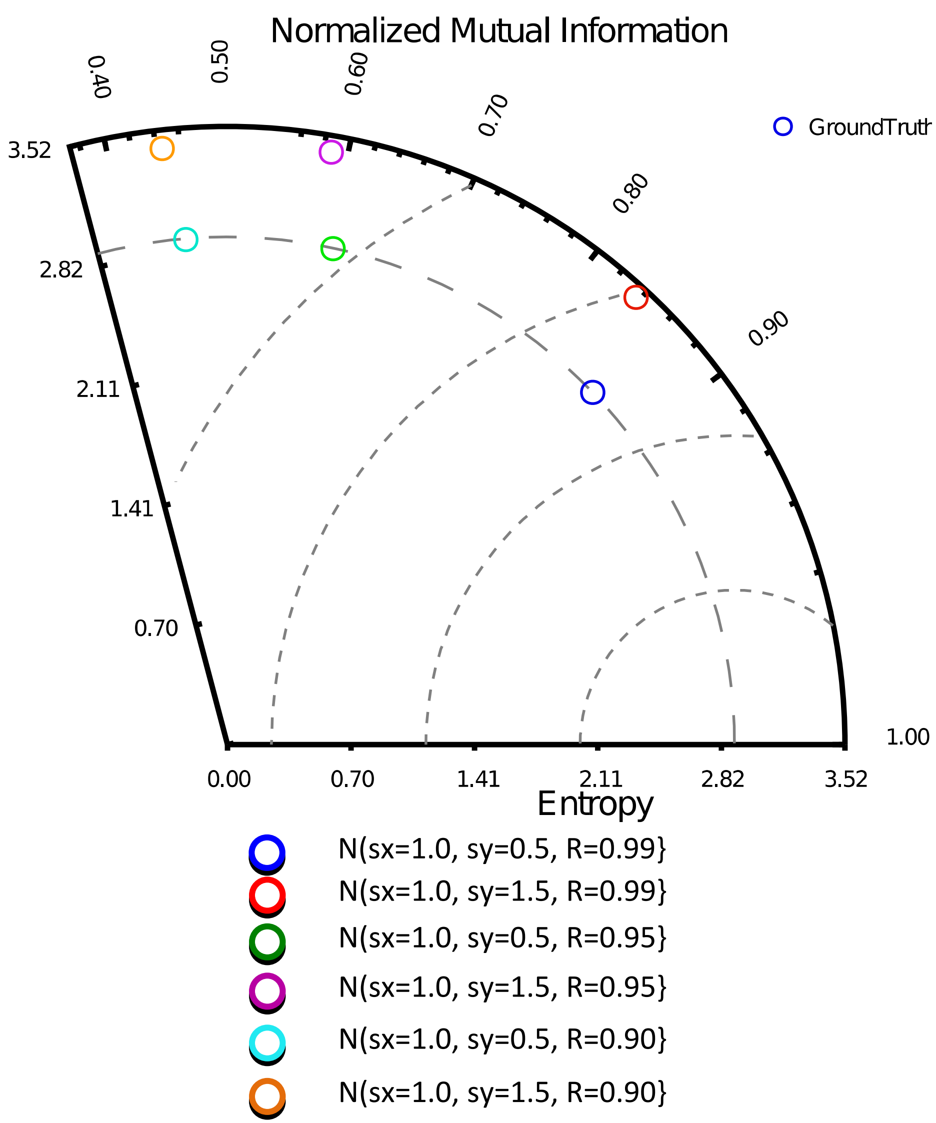
Outliers have a significant impact on correlation

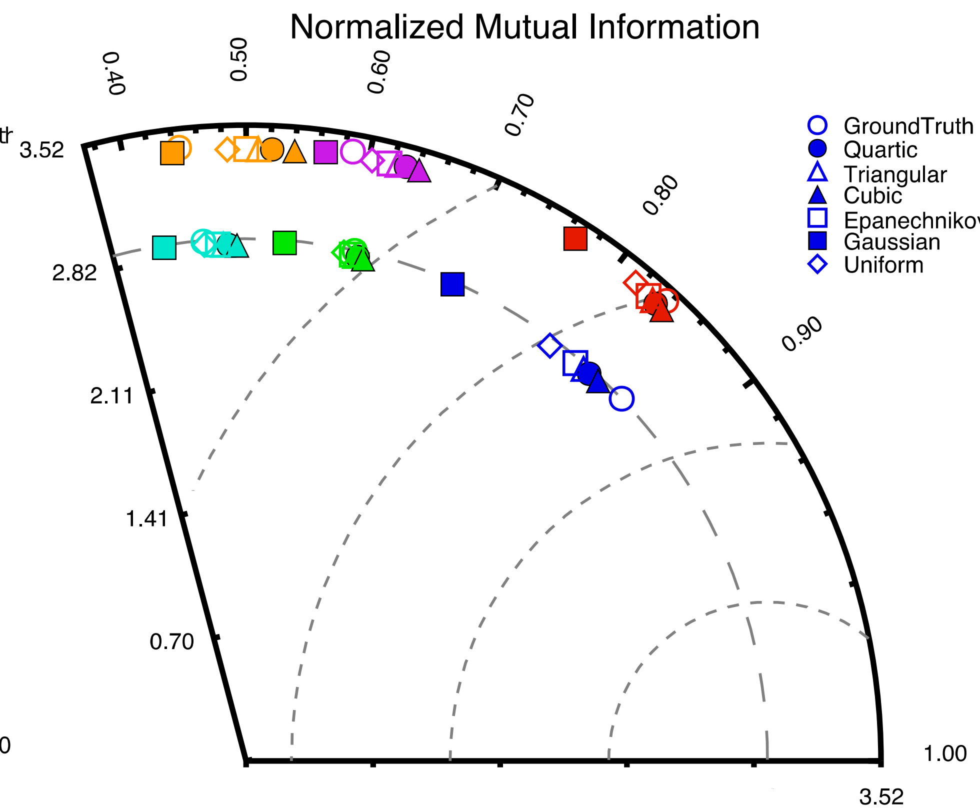


The information in both the "clean" and "dirty" distributions is essentially the same



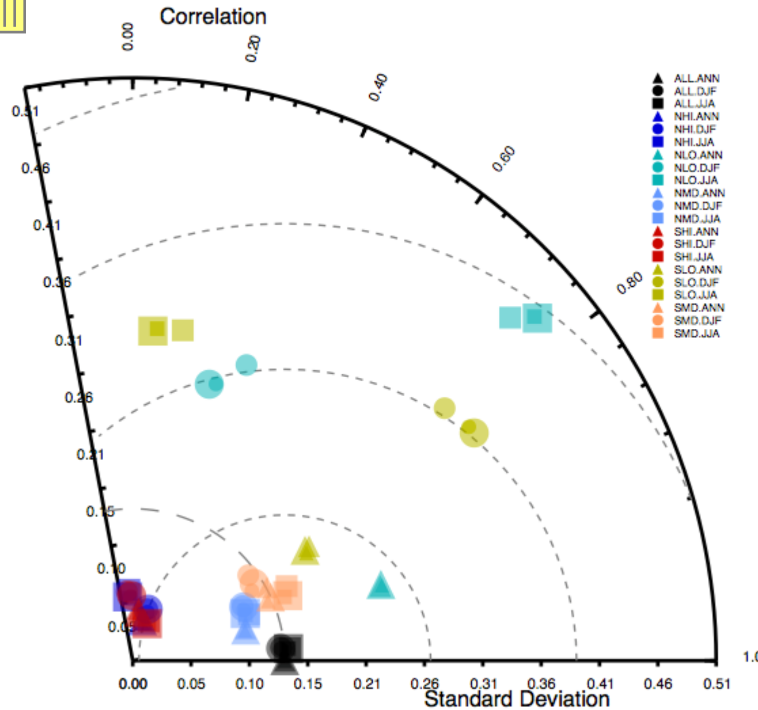
Computing Entropy and Mutual Information *may* require estimation of underlying probability functions





Although there are differences, relative distances are consistent for each choice of kernel

Uncertainty Quantification in Climate Simulations

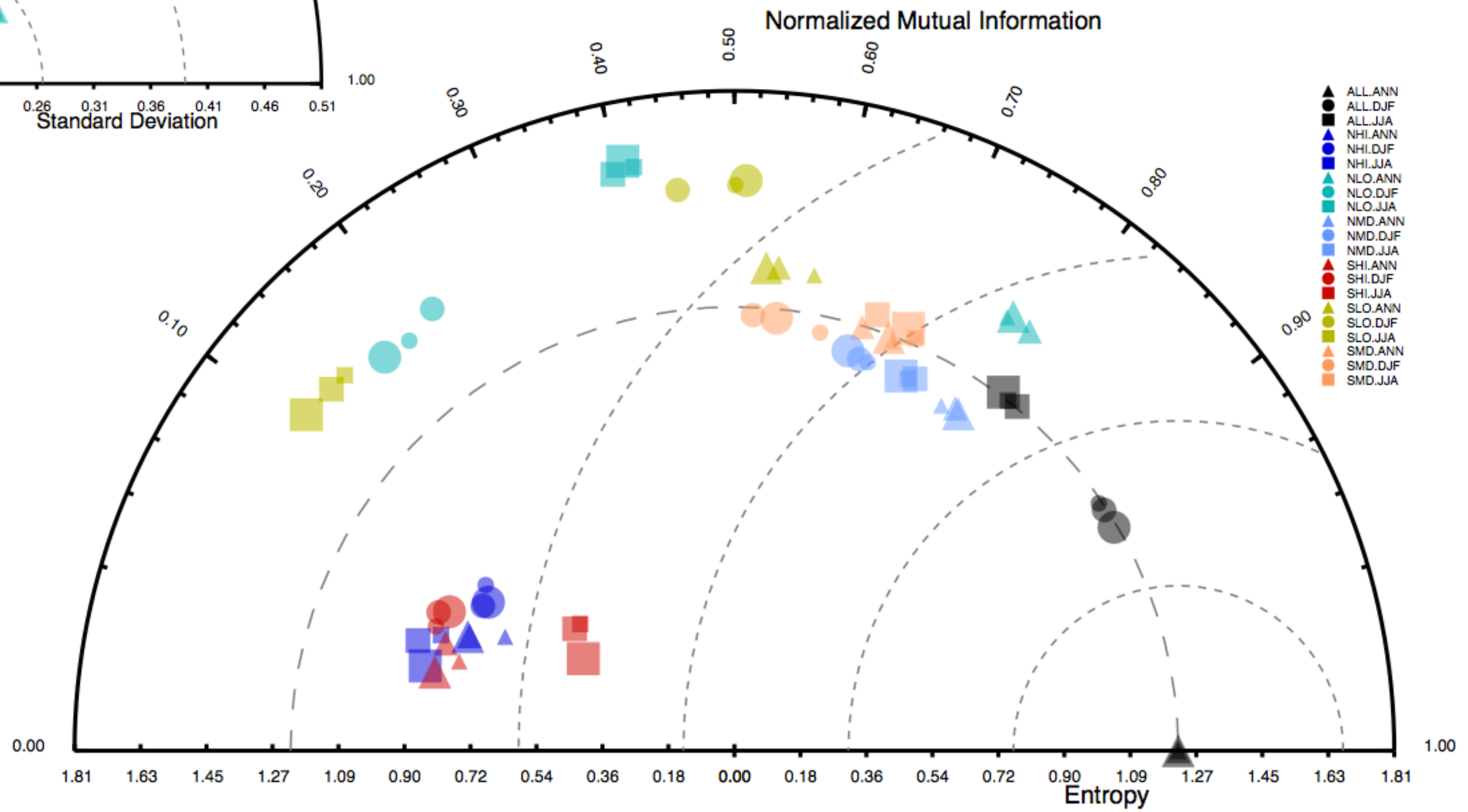


Precipitation average

7 Zonal averages (color)

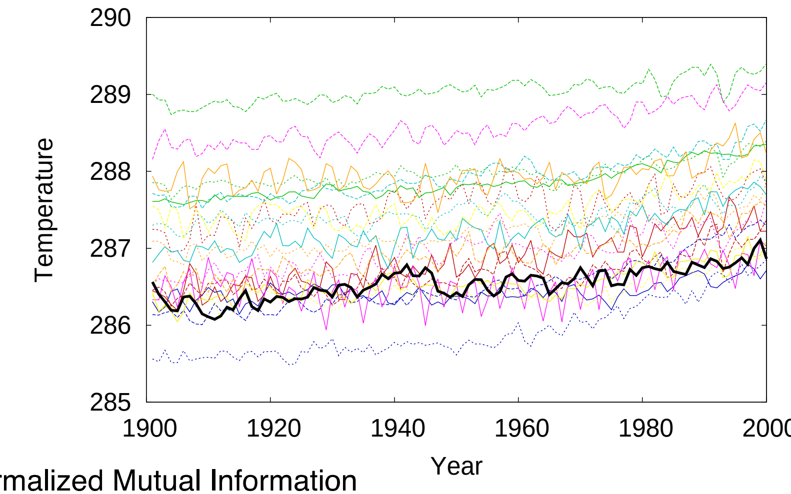
3 Temporal averages (shape)

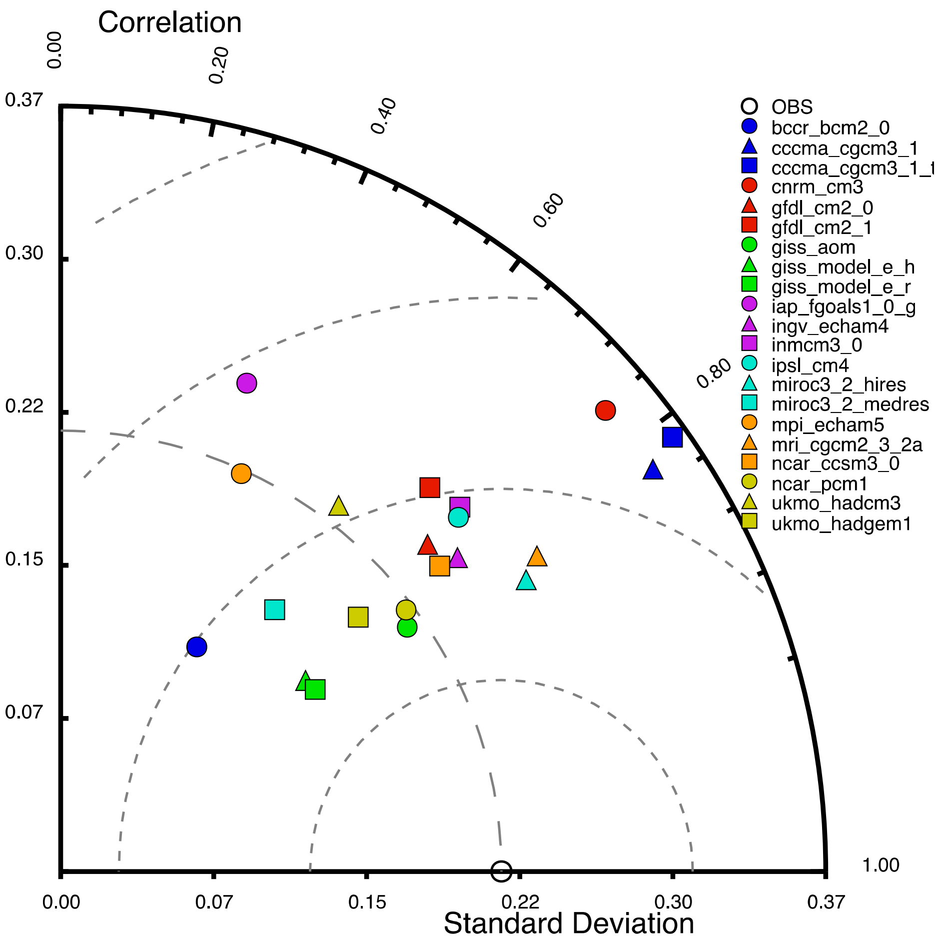
3 different ensemble sets (size)

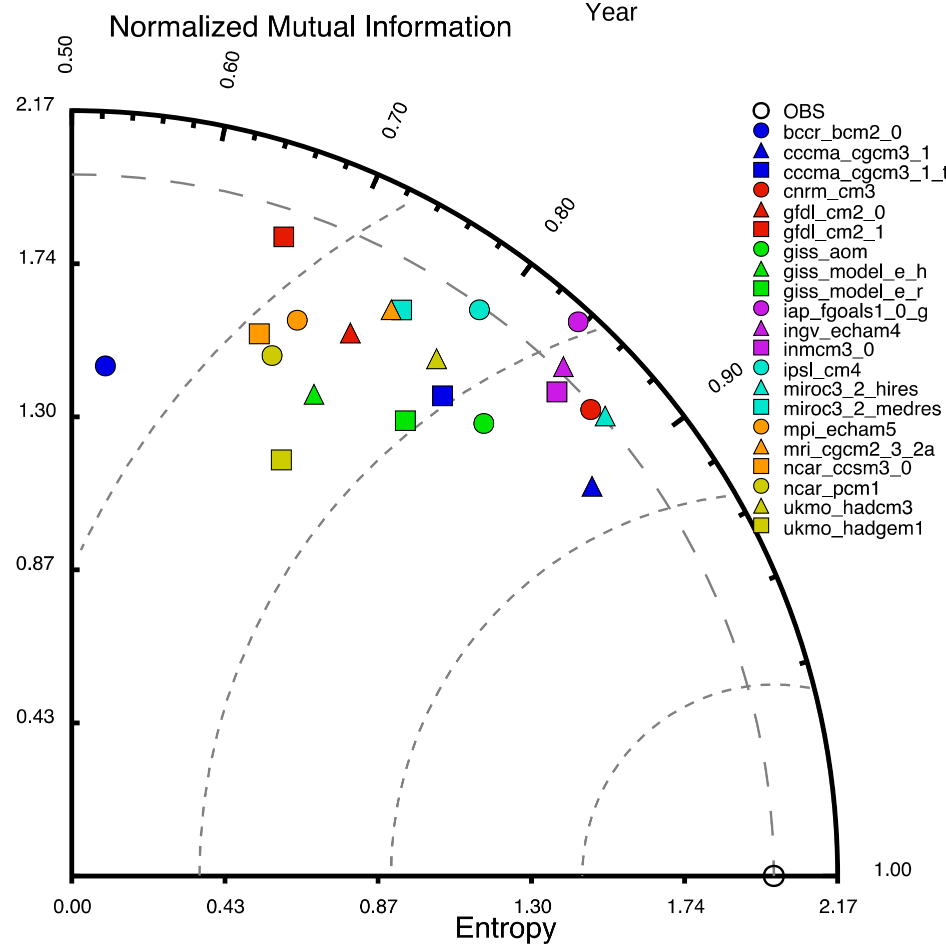


Intercomparison Studies

Annual mean temperature 1900-2000







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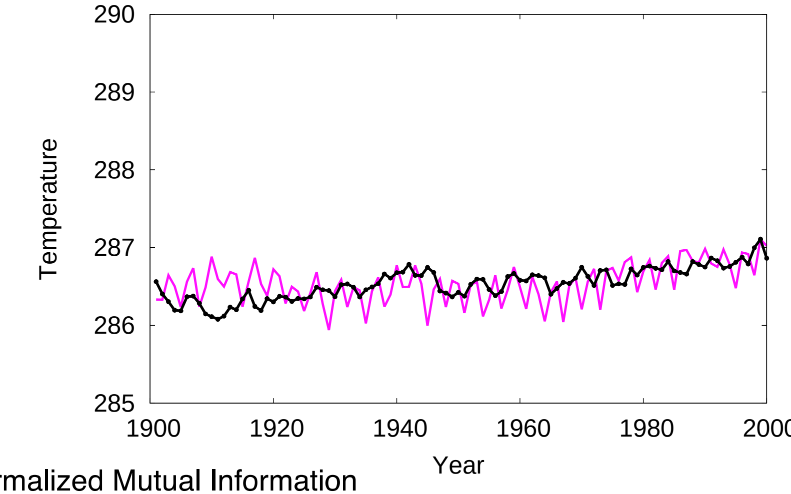
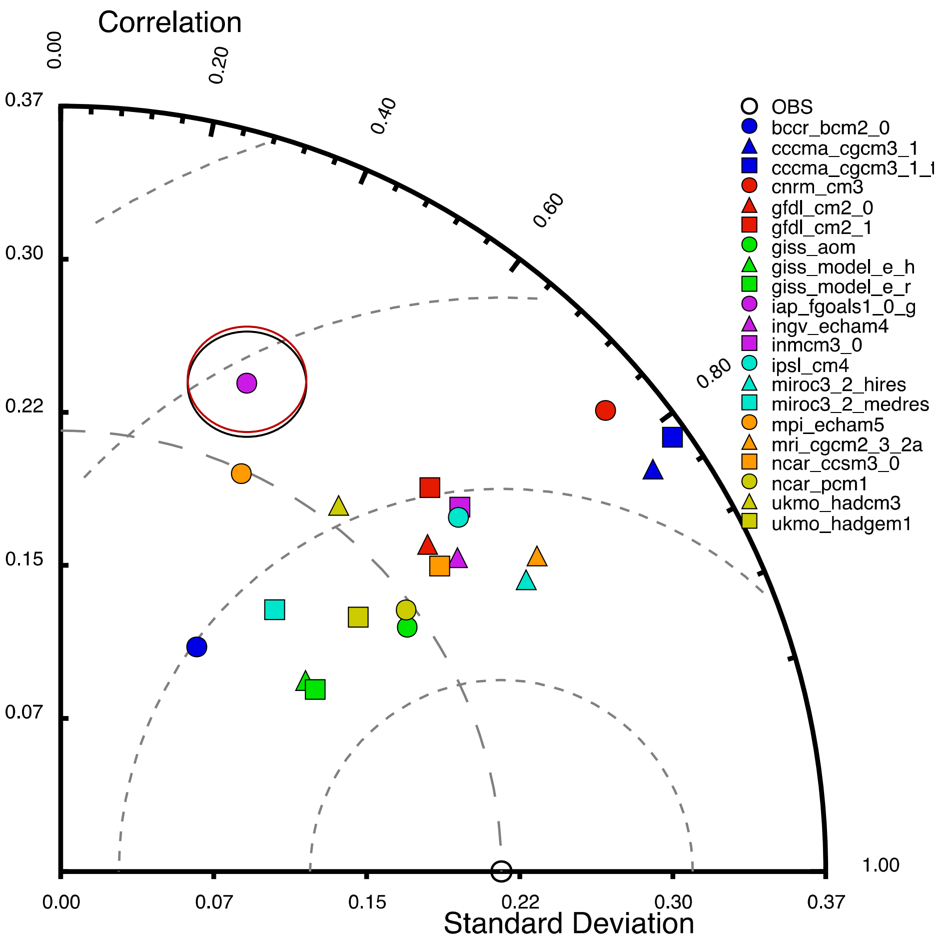
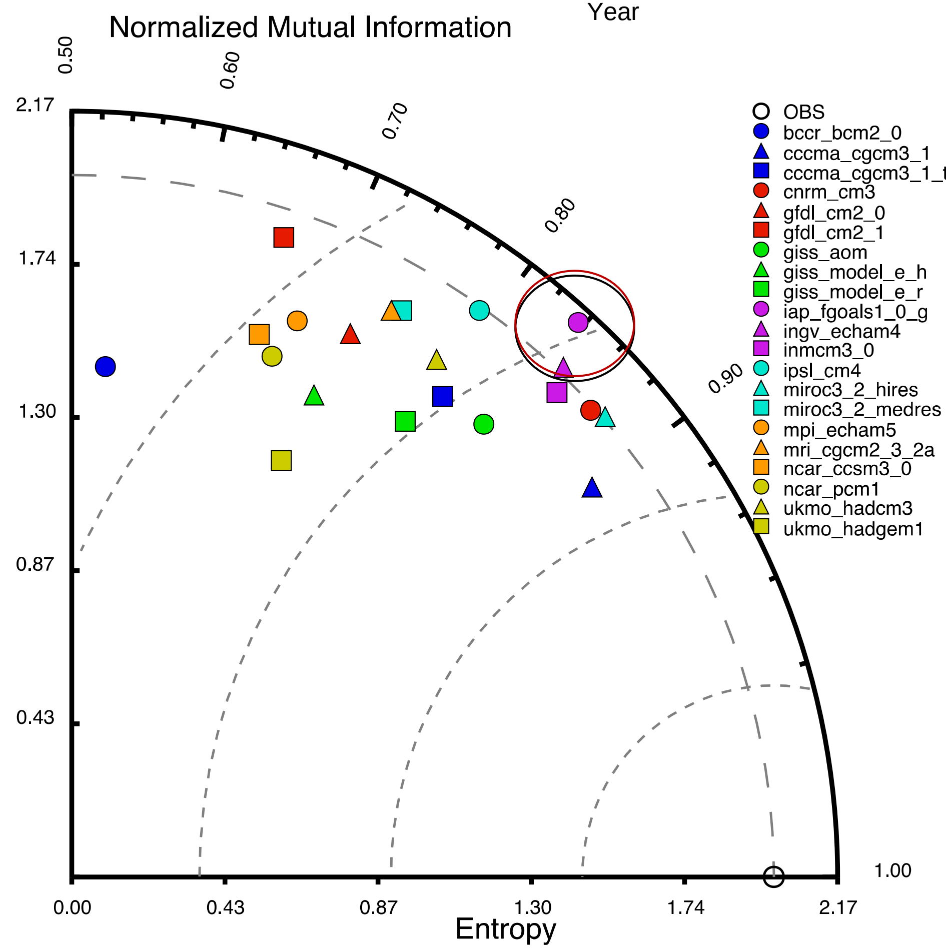
Annual mean temperature 1900-2000







Intercomparison Studies

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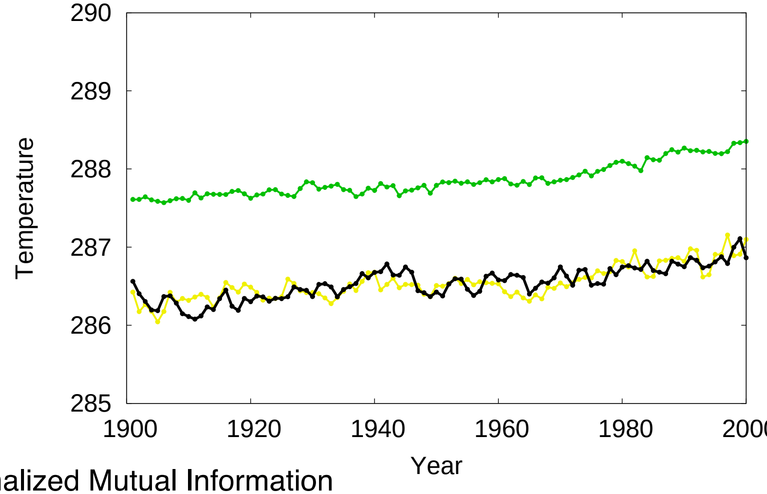
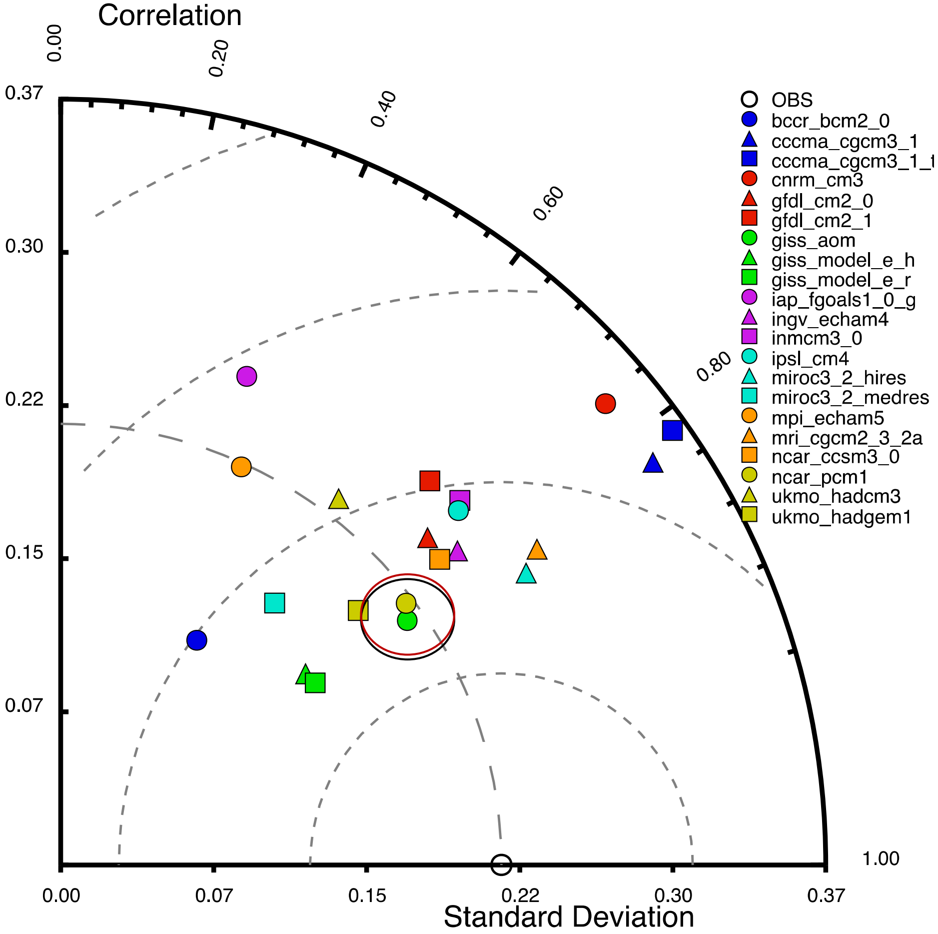
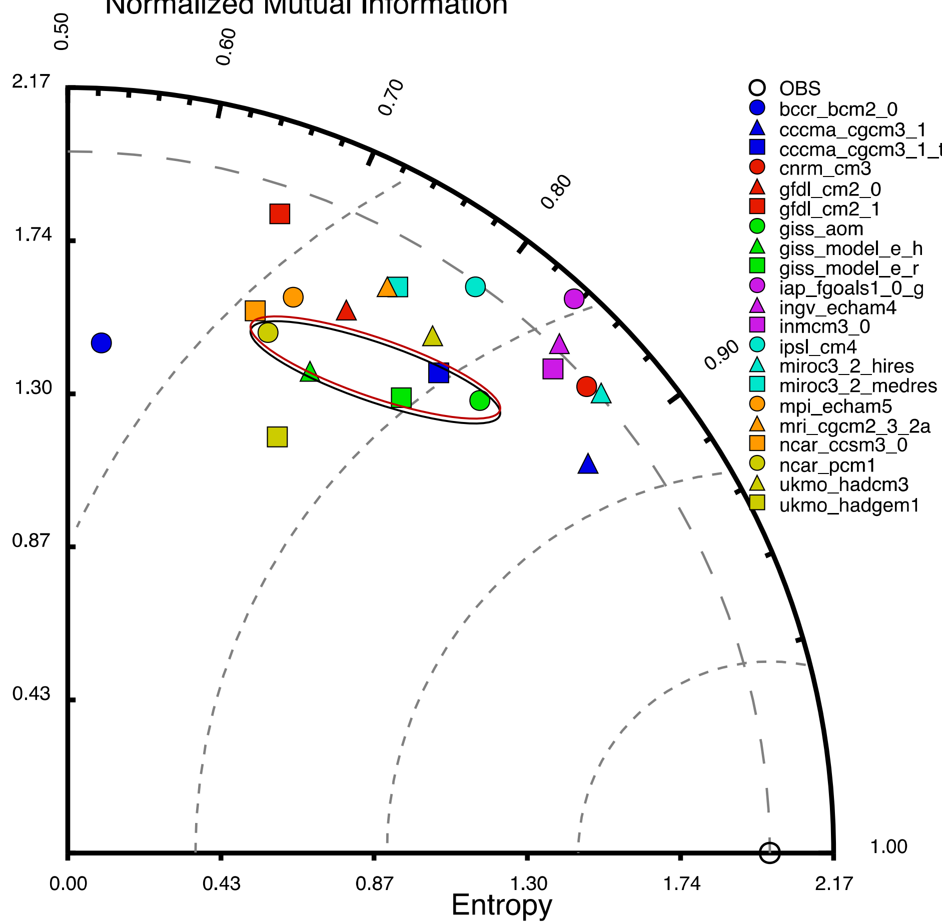
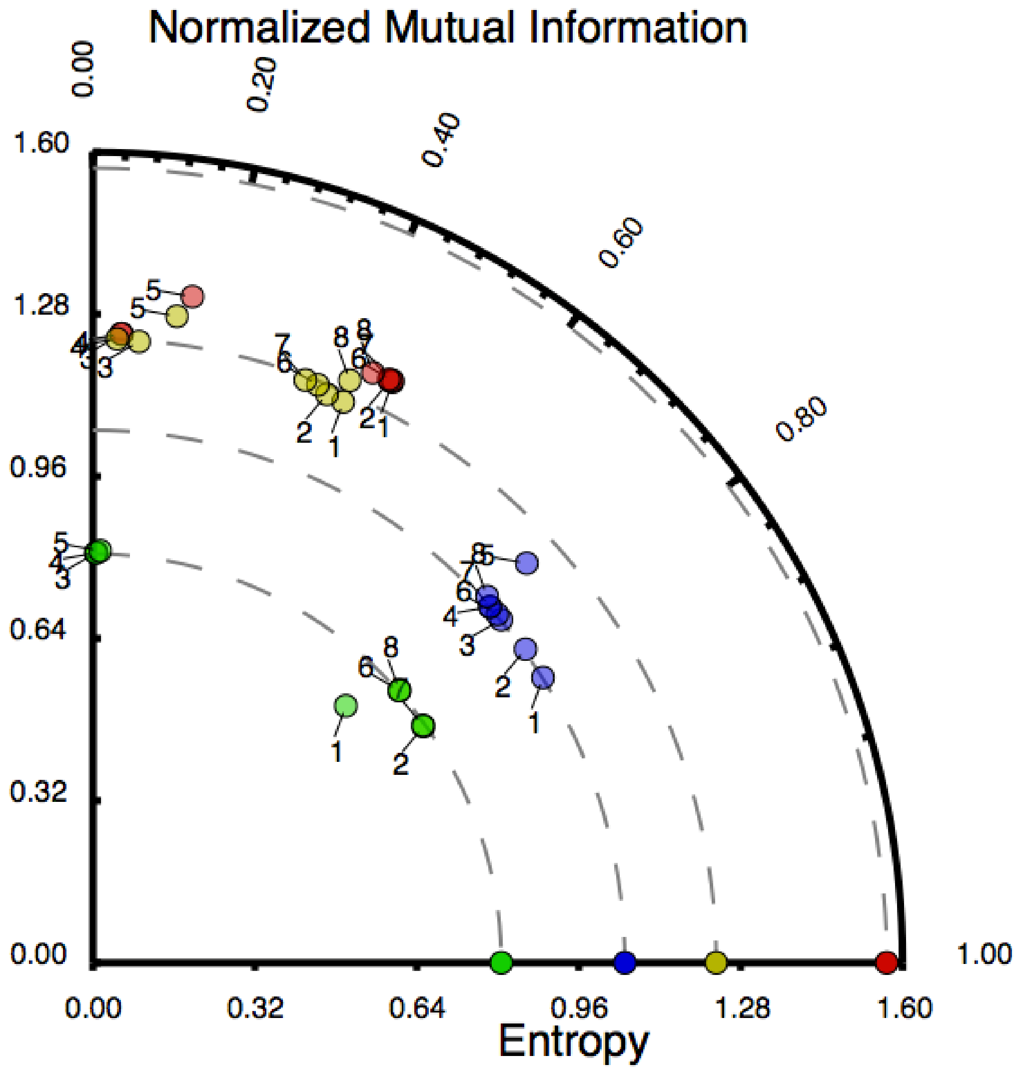






MID applies to discrete data: useful when comparing Clustering Results



- Summarize study in clustering [Filippone et al. 2009]
- 8 different methods
- 4 classification problems

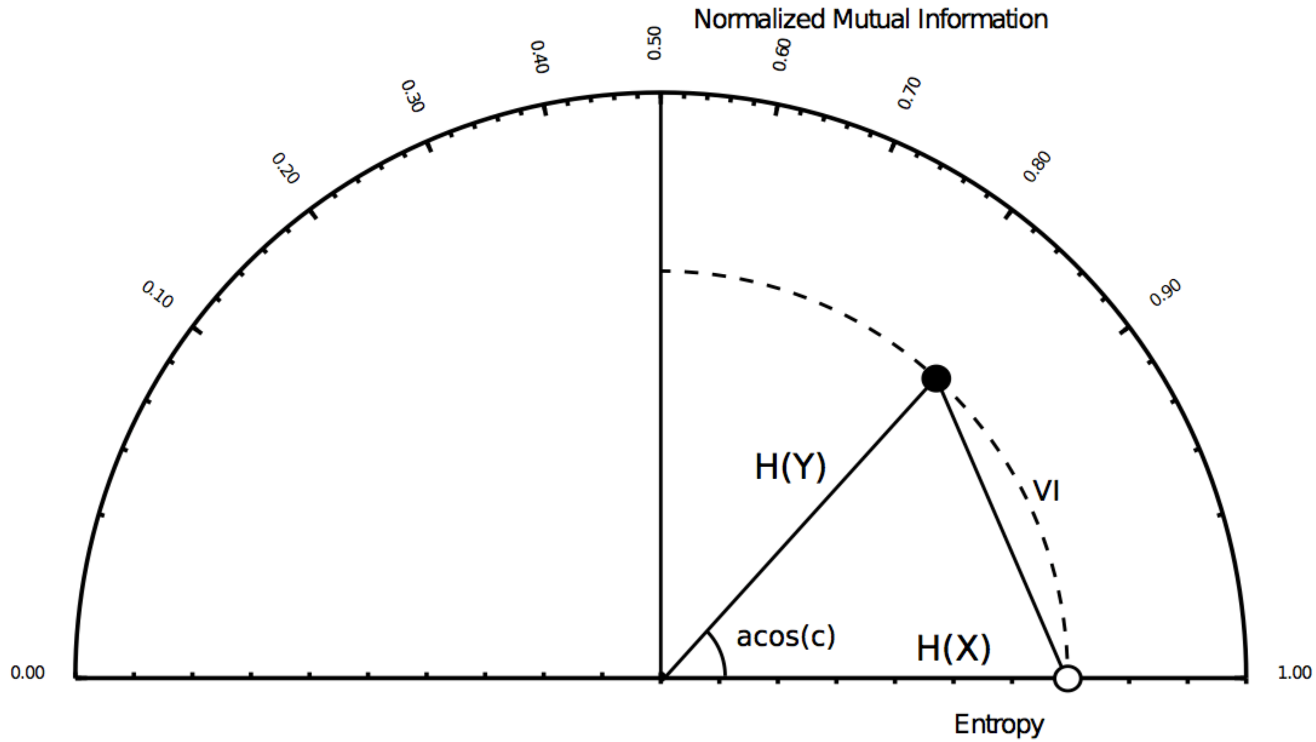
Concluding Remarks

- Taylor diagram:
  - easy to compute.
  - Well understood in geophysical sciences, climate.

- MI diagram:
  - Counterpart using information theory.
  - requires an estimation step that may introduce additional uncertainties.
  - extends nicely to categorical data, multi-variate distributions.
  - exposes non-linearities, difficult to see via (linear) correlation.

- More informed decisions when combining both diagrams.

Thanks!



Questions?